Quantum-to-classical transition and stochastic inflation

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Summary

- Stochastic inflation allows the computation of non-linear perturbations for PBH formation
- Quantum-to-classical transition needs to be understood for good predictions
- Squeezing is one way to measure the level of classicality

Stochastic inflation

In cosmic inflation, the expansion of the universe accelerates, driven by the inflaton ϕ. We divide it into long and short wavelength parts,

$$\phi = \int_{k<k_c} \frac{\mathrm{d}^3k}{(2\pi)^{\frac{3}{2}}}\phi_{\mathbf{k}}(t)e^{-i\mathbf{k}\cdot\mathbf{x}} + \int_{k>k_c} \frac{\mathrm{d}^3k}{(2\pi)^{\frac{3}{2}}}\phi_{\mathbf{k}}(t)e^{-i\mathbf{k}\cdot\mathbf{x}} \equiv \bar{\phi} + \delta\phi\,, \quad (1)$$

separated by the **coarse-graining scale** $k_c \equiv \sigma a H$, $\sigma < 1$. We evolve the short-wavelength perturbations linearly in the local background:

$$\partial_N^2\delta\phi_{\mathbf{k}} + (3-\epsilon_1)\,\partial_N\phi_{\mathbf{k}} + \omega_k^2\delta\phi_{\mathbf{k}} = 0\,. \quad (2)$$

The long-wavelength part behaves like a local FRW universe:

$$\begin{aligned}\partial_N\bar{\phi} &= \bar{\pi} + \xi_\phi\,,\\ \partial_N\bar{\pi} &= -(3-\epsilon_1)\,\bar{\pi} - H^{-2}V_{,\bar{\phi}} + \xi_\pi\,,\\ H^2 &= 2V/(6-\bar{\pi}^2)\,.\end{aligned} \quad (3)$$

Here ξ_ϕ, ξ_π describe the **stochastic noise** from short-wavelength perturbations that cross the coarse-graining scale, leading to **stochastic inflation**.

The linear perturbations are treated quantum mechanically, while the coarse-grained quantities evolve classically and include non-perturbative interactions. This allows us to probe large perturbations for which non-linearity is important. For the description to be valid, the perturbations need to undergo a **quantum-to-classical transition** near k_c.

Primordial black holes from large perturbations

Stochastic inflation can be used to predict the abundance of primordial black holes (PBHs). Using the so-called ΔN formalism, we convert the coarse-grained field perturbations into coarse-grained comoving curvature perturbations $\mathcal{R}_c$. Solving Eqs. (2), (3), either analytically under simplifying assumptions or numerically in full realization by realization, we can derive a **probability distribution for** $\mathcal{R}_c$. Large perturbations exceeding a threshold will collapse into PBHs after inflation when the corresponding scale re-enters the horizon. See Fig. 1 for details.

Role of the quantum-to-classical transformation

We assume the quantum-to-classical transformation to happen **instantaneously** at scale k_c. This simplifies the computation. A more refined description of the process may affect the PBH predictions.

In the current formalism, the value of σ in $k_c = \sigma a H$ has a mild impact on the results. We want a large σ to include as much of the non-linear interactions as possible; but σ needs to be small enough to allow the quantum modes to classicalize. We measure the classicality of the modes by their **squeezing**.

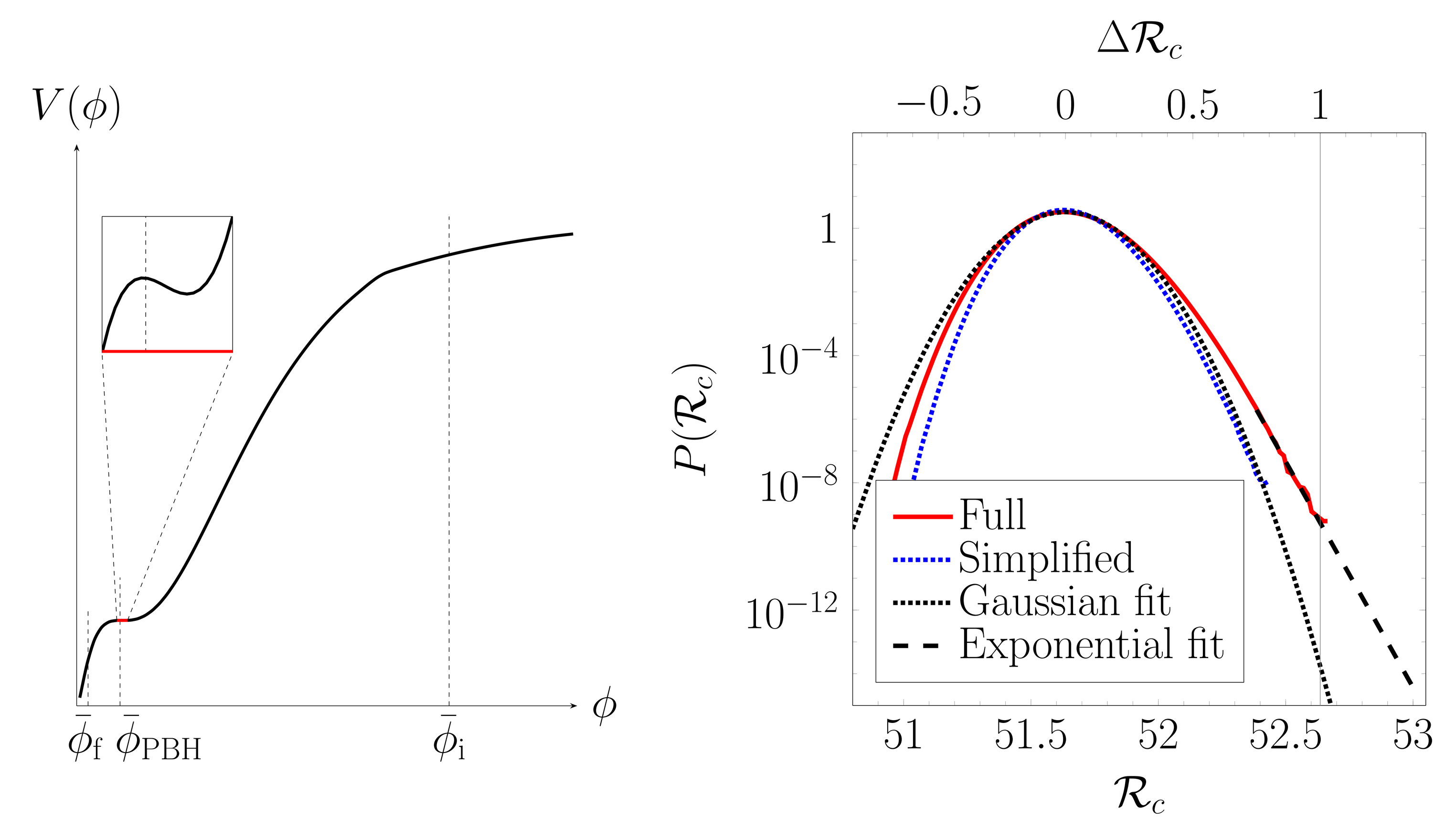

Fig. 1: *Left:* A typical inflaton potential for PBH production: rolling over a local maximum boosts the perturbations. *Right:* The $\mathcal{R}_c$ distribution for a typical PBH scale in the same model, computed numerically. The **exponential tail** is out of reach for perturbative techniques and boosts PBH production significantly compared to a naive Gaussian approximation. Results from arXiv:2012.06551.

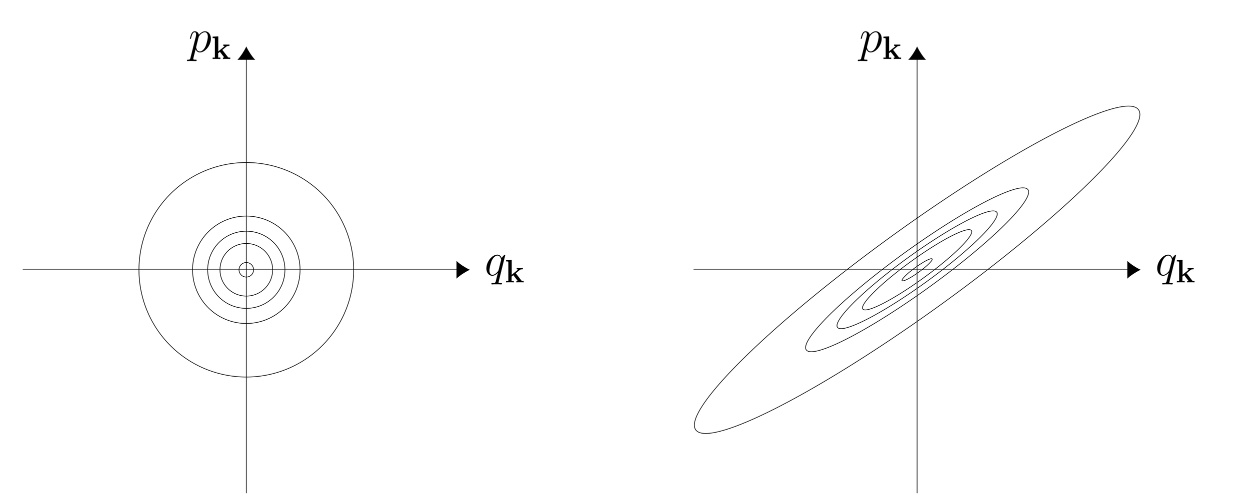

Fig. 2: Contours for the Wigner function of the inflaton mode variables $q_{\mathbf{k}}$, $p_{\mathbf{k}}$ from (5), in the initial minimum uncertainty vacuum state (*left*) and later on in a squeezed state (*right*).

Squeezing

We quantize the short-wavelength perturbations in the standard way:

$$\hat{\phi}_{\mathbf{k}} \equiv \phi_k\hat{a}_{\mathbf{k}} + \phi_k^*\hat{a}_{-\mathbf{k}}^\dagger\,, \qquad \left[\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}^\dagger\right] = \delta_{\mathbf{k},\mathbf{k}'}\,. \quad (4)$$

The corresponding canonical variables $\hat{q}_{\mathbf{k}}, \hat{p}_{\mathbf{k}}$, obeying $[\hat{q}_{\mathbf{k}}, \hat{p}_{\mathbf{k}}] = i$ (with $\hbar = 1$), read

$$\hat{q}_{\mathbf{k}} = k^{1/2}a\hat{\phi}_{\mathbf{k}}\,, \qquad \hat{p}_{\mathbf{k}} = k^{-1/2}a^2H\partial_N\hat{\phi}_{\mathbf{k}}\,. \quad (5)$$

$k \gg aH$:
The modes start in the Bunch–Davies vacuum state of a minimum uncertainty wave packet: $\Delta q_{\mathbf{k}} = \Delta p_{\mathbf{k}} = 1/\sqrt{2}$ (see Fig. 2).

$k \ll aH$:
Around horizon exit, the modes get **squeezed**: the uncertainty grows in both directions, $\Delta q_{\mathbf{k}}$, $\Delta p_{\mathbf{k}} \gg 1$ (see Fig. 2). The canonical commutator can be approximated as zero when computing correlation functions; the 'quantumness' of the probability distribution is suppressed. We identify the squeezed quantum probability distribution with a classical one, used to produce the noise terms ξ_ϕ, ξ_π in Eq. (3).

Threshold for classicality

The squeezing of the state can be measured by

$$\cosh(2r_{\mathbf{k}}) = (\Delta q_{\mathbf{k}})^2 + (\Delta p_{\mathbf{k}})^2\,. \quad (6)$$

We call a mode classical once $\cosh(2r_{\mathbf{k}}) > 100$, and choose σ in $k_c = \sigma a H$ so that this is true for all relevant modes as they exit the coarse-graining scale. Typically this means $\sigma \approx 0.01$.

However, $\cosh(2r_{\mathbf{k}})$ is not an ideal measure of classicality. For example, it can momentarily decrease during ultra-slow-roll inflation. Improved practical measures of classicality, based on e.g. decoherence, are needed to better understand the nuances of stochastic inflation.

Open questions

- How to better measure the classicality of perturbations in stochastic inflation?
- How to describe the quantum-to-classical transition in a precise way?