Higgs inflation and quantum corrections

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Contents

- Basics of Higgs inflation
- Recent developments: quantum corrections
- Two case studies
  - Hilltop Higgs inflation
  - (Near-)critical point and primordial black holes

Motivation

- Cosmic inflation: explains homogeneity, isotropy, flatness of the universe; predicts CMB anisotropies
- Higgs inflation: simple model; SM Higgs is the inflaton, no new fields beyond the SM
  - Only non-minimal coupling to gravity ξ needs to be added

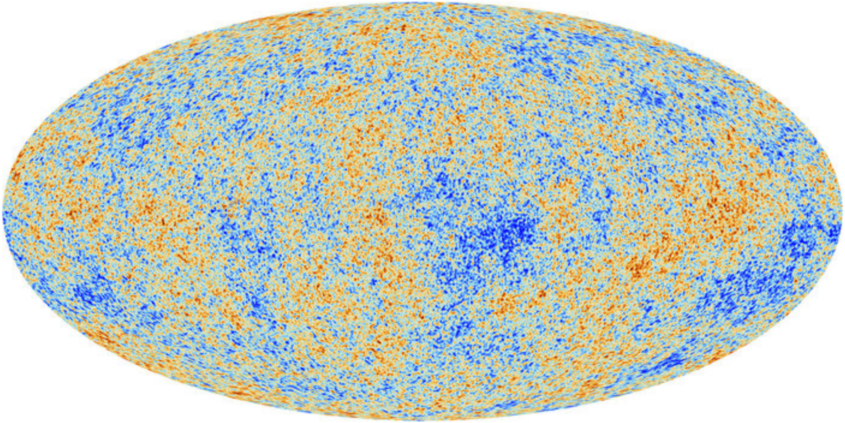
Cosmic inflation

- Inflaton field ϕ dominates energy density of the universe: accelerating expansion

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0\,,$$
$$3H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

- Dynamics determined by the potential $V(\phi)$
- Perturbations on top of the homogeneous field

Cosmic inflation and CMB observations



Cosmic inflation and CMB observations

- CMB scale exits the Hubble radius 50 e-folds before the end of inflation. There:

$$A_s = 2.1 \times 10^{-9}, \qquad n_s = 0.9625 \pm 0.0048,$$
$$\alpha_s = 0.002 \pm 0.010, \quad \beta_s = 0.010 \pm 0.013,$$
$$r < 0.079$$

CMB predictions

- Slow-roll approximation:

$$3H\dot{\phi} \approx -V'(\phi)\,, \qquad 3H^2 \approx V(\phi)$$

- There:

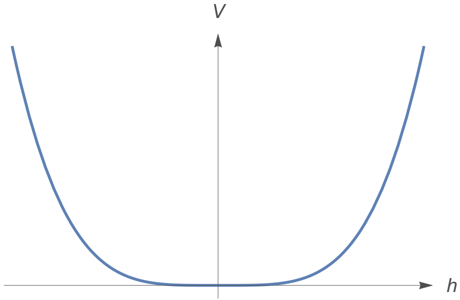
$$N = \int \frac{d\phi}{\sqrt{2\epsilon_V}}\,, \qquad A_s = \frac{V}{24\pi^2\epsilon_V}\,,$$

$$n_s = 1 - 6\epsilon_V + 2\eta_V\,, \qquad r = 16\epsilon_V\,,$$

$$\epsilon_V \equiv \frac{1}{2}\left(\frac{V'}{V}\right)^2, \qquad \eta_V \equiv \frac{V''}{V}$$

Prelude: Minimal Higgs inflation

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}M^2R + \frac{1}{2}g^{\mu\nu}\partial_\mu h\partial_\nu h - \frac{\lambda}{4}h^4\right]$$



- Problem: even with best fit, need $\lambda \sim 10^{-13}$ and get $r > 0.1$

Non-minimal Higgs inflation

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}\left(M^2 + \xi h^2\right)R + \frac{1}{2}g^{\mu\nu}\partial_\mu h\partial_\nu h - \frac{\lambda}{4}h^4\right]$$

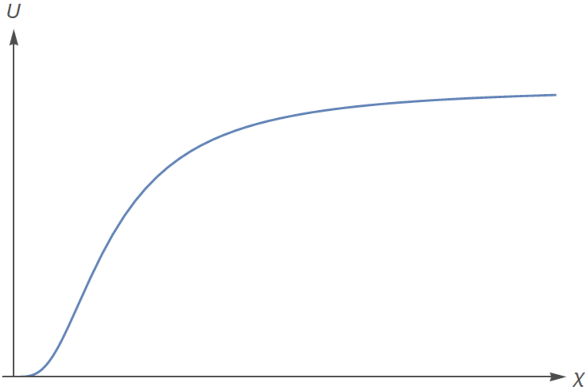
- First studied for SM Higgs in [0710.3755]
- Standard procedure: Weyl transformation

$$g_{E\mu\nu} = g_{\mu\nu}\left(1 + \frac{\xi h^2}{M^2}\right), \quad \frac{dh}{d\chi} = \frac{1 + \xi h^2}{\sqrt{1 + \xi h^2 + 6\xi^2 h^2}}$$

$$S_E = \int d^4x\sqrt{-g_E}\left[-\frac{1}{2}M^2 R_E + \frac{1}{2}g_{E\mu\nu}\partial^\mu\chi\partial^\nu\chi - U(\chi)\right]$$

Non-minimal Higgs inflation

- Einstein frame potential:



$$V = \frac{\lambda}{4} F^4[h(\chi)] , \qquad F(h) \equiv \frac{h}{\sqrt{1 + \xi h^2}} \approx \frac{1}{\sqrt{\xi}} \left(1 - e^{-\sqrt{2/3}\chi} \right)^{1/2}$$

Non-minimal Higgs inflation

- CMB predictions fit the observations:

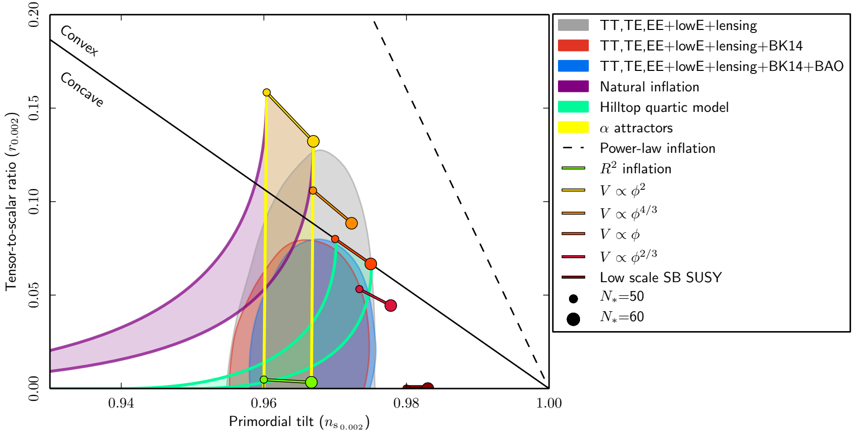
$$A_s = \frac{\lambda N^2}{72\pi^2\xi^2}\,,$$

$$n_s = 1 - \frac{2}{N}\,, \quad r = \frac{12}{N^2}$$

- For $N \sim 50$, $\xi = 800\sqrt{\lambda}N$:

$$n_s \approx 0.96\,, \quad r \approx 4.8 \times 10^{-3}$$

Models of inflation vs Planck



Quantum corrections

- Add loop corrections:

$$V_{1-\mathrm{loop}}(\phi) \sim m^4(\phi) \ln \frac{m^2(\phi)}{\mu^2}$$

- Run the couplings to μ value that makes the corrections small:

$$\frac{d\alpha}{d \ln \mu} = \beta$$

Quantum corrections

- Problem: model is not renormalizable!
  - Jordan frame: non-renormalizable gravity
  - Einstein frame: non-polynomial tree-level potential V
- Approximation: Chiral standard model
  - Inflationary region: $V \to const.$
  - Higgs field decouples from others: "Chiral SM"
  - NOT renormalizable, but at each order, only a finite number of new couplings, and they are known

Quantum corrections: "Recipe"

- Potential: $V = V_{tree} + V_{1-\mathrm{loop}}$,

$$V_{1-\mathrm{loop}} = \frac{6m_W^4}{64\pi^2}\left(\ln\frac{m_W^2}{\mu^2} - \frac{5}{6}\right) + \frac{3m_Z^4}{64\pi^2}\left(\ln\frac{m_Z^2}{\mu^2} - \frac{5}{6}\right) - \frac{3m_t^4}{16\pi^2}\left(\ln\frac{m_t^2}{\mu^2} - \frac{3}{2}\right),$$

$$m_W^2 = \frac{g^2F^2}{4}, \qquad m_Z^2 = \frac{(g^2 + g'^2)F^2}{4}, \qquad m_t^2 = \frac{y_t^2F^2}{2},$$

Quantum corrections: "Recipe"

- Running:

$$16\pi^2\beta_\lambda = -6y_t^4 + \frac{3}{8}\left(2g^4 + \left[g^2 + g'^2\right]^2\right),$$

$$16\pi^2\beta_{y_t} = y_t\left(\frac{9}{2}y_t^2 - \frac{9}{4}g^2 - \frac{17}{12}g'^2 - 8g_s^2\right),$$

$$16\pi^2\beta_g = -\frac{19}{6}g^3, \quad 16\pi^2\beta_{g'} = \frac{41}{6}g'^3, \quad 16\pi^2\beta_{g_s} = -7g_s^3$$

- Renormalization scale $\mu \sim \gamma F$

Connection to accelerator physics

- $h \ll 1/\xi$: Standard Model
- $h \gg 1/\sqrt{\xi}$: Chiral Standard Model
- $1/\sqrt{\xi} \ll h \ll 1/\xi$: Corrections out of control!

- Parametrize unknown physics by effective "jumps" in λ and y_t

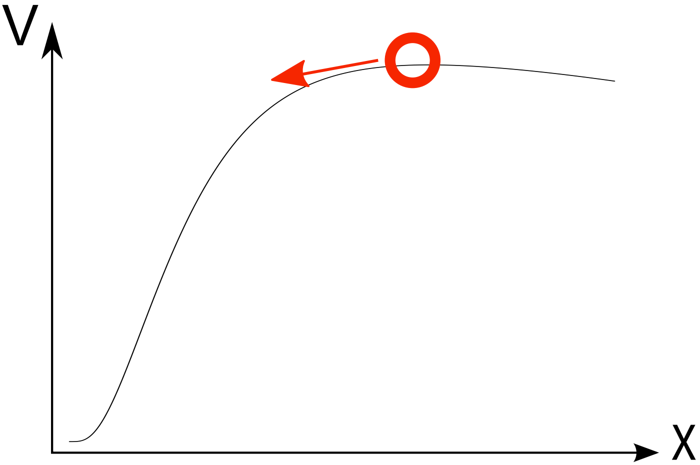
Quantum corrections in Higgs inflation

- Many studies
- Conclusions: in most cases, no big effect on CMB observables
- EXCEPTION: when parameters fine-tuned so that a feature is formed in the potential

Caveats

- New physics (new operators) at big h?
- Choice of frames (choice of renormalization scale)?
- Breakdown of tree-level unitarity?
- Unstable EW vacuum?

Special case: Hilltop



Hilltop inflation

- Quantum corrections produce a local maximum into the potential [1802.09299]
- Analytic calculation: write

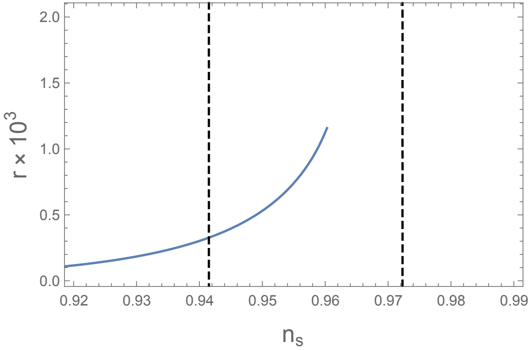
$$V = \frac{\lambda_{\text{eff}}}{4} F^4 , \qquad V' = \frac{F^3 F'}{4} (4\lambda_{\text{eff}} + \beta_{\text{eff}}) ,$$

$$\lambda_{\text{eff}} = \lambda_0 - 4\lambda_0 \ln \frac{\mu}{\mu_0} , \quad \mu = \gamma F$$

- Expand in $\delta \equiv 1/(\xi h^2)$, solve slow-roll equations analytically

Hilltop inflation

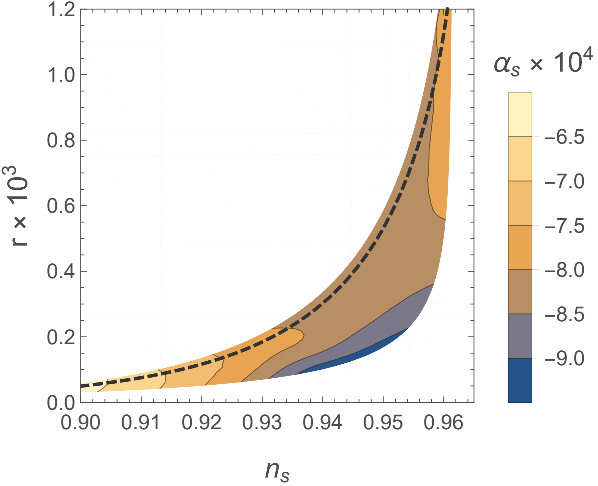
- Analytical approximation: predictions



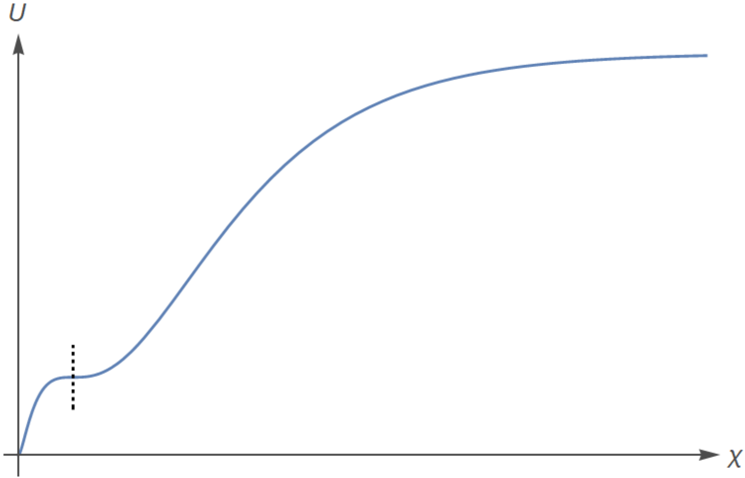
- Note: $r < \frac{3}{N^2}$, smaller by at least a factor of four compared to tree level result

Hilltop inflation

- Numerical scan:



Special case: Critical point



Critical point inflation

- Chance for production of primordial black holes (PBH)
  - Dark matter?
- Slow roll: perturbations amplified for $\epsilon_V \to 0$, since

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{V}{24\pi^2\epsilon_V}$$

Critical point inflation

- Near the critical point: slow-roll broken!
- Instead, ultra slow roll (USR): $V' = 0$, so

$$\ddot{\chi} + 3H\dot{\chi} = 0$$

- Need to calculate perturbations numerically:

$$\mu_k'' + \left(k^2 - \frac{z''}{z}\right)\mu_k = 0\,, \qquad z \equiv a\frac{\dot{\phi}}{H}\,,$$

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}\frac{|\mu_k|^2}{z^2}$$

PBH formation

- PBH fraction:

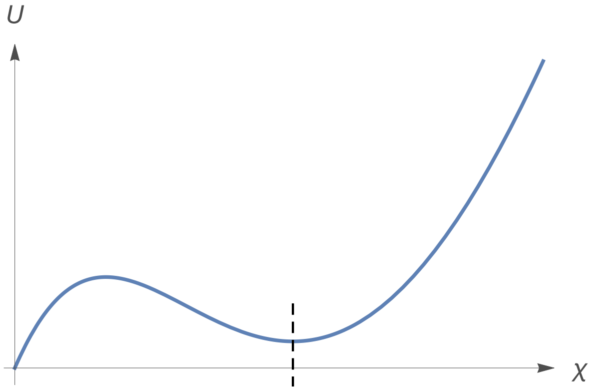
$$\Omega_{\mathrm{PBH\,eq}} \propto e^{-\frac{\zeta_c^2}{2\mathcal{P}_{\mathcal{R}}(k)}+\Delta N}$$

- To be significant, need $\mathcal{P}_{\mathcal{R}}(k) \gtrsim 10^{-4}$
- PBH mass:

$$M_{\mathrm{PBH}} = \gamma \frac{4\pi}{3} H^{-3} \rho \approx 2 \times 10^{15} \times e^{-2\Delta N} M_{\odot}$$

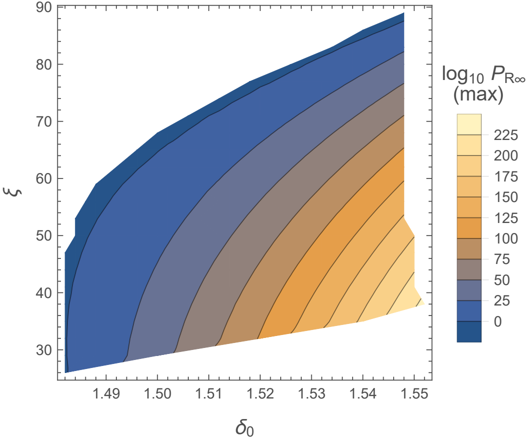
PBH formation

- Most efficient production for a potential with a local minimum:



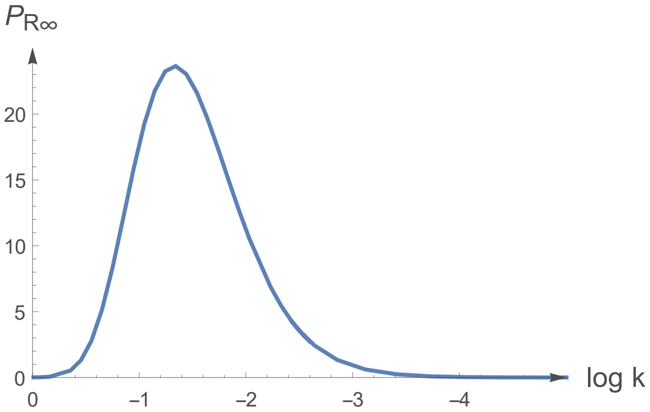
PBH formation

- Numerical scan: [1810.12608]

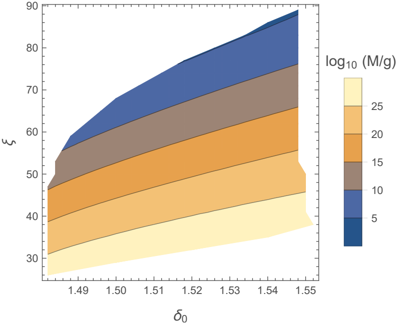


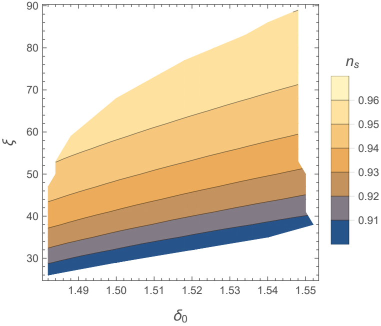
- Efficient PBH production!

PBH numerics

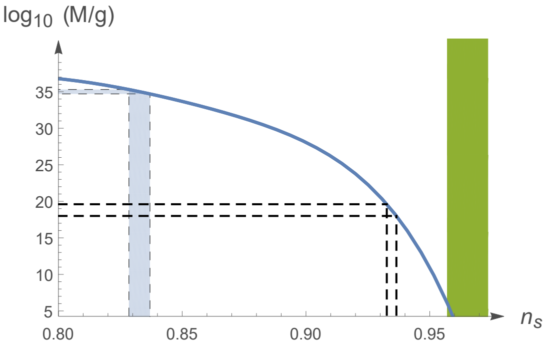


PBH numerics





PBH mass versus n_s



- Problem: need $M < 10^6$ g
- Evaporation by Hawking radiation!
- Planck mass relics?

Summary

- Higgs inflation: a versatile, attractive model of cosmic inflation
- Quantum corrections to inflaton potential can affect cosmology
- Hilltop Higgs inflation: r smaller that at tree level by a factor of four or more
- Near critical point Higgs inflation: can only produce small PBHs (dark matter only as evaporation relics)

PBH numerics