A numerical approach to stochastic inflation and primordial black holes

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Eemeli Tomberg, NICPB Tallinn
eemeli.tomberg@kbfi.ee

Based on 2012.06551, in collaboration with D. Figueroa, S. Raatikainen, S. Räsänen

Concepts

Cosmic inflation

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Cosmological perturbations

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Cosmic inflation

Cosmological perturbations

Primordial black holes

Concepts

Stochastic inflation

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Stochastic inflation

- Includes non-linear effects

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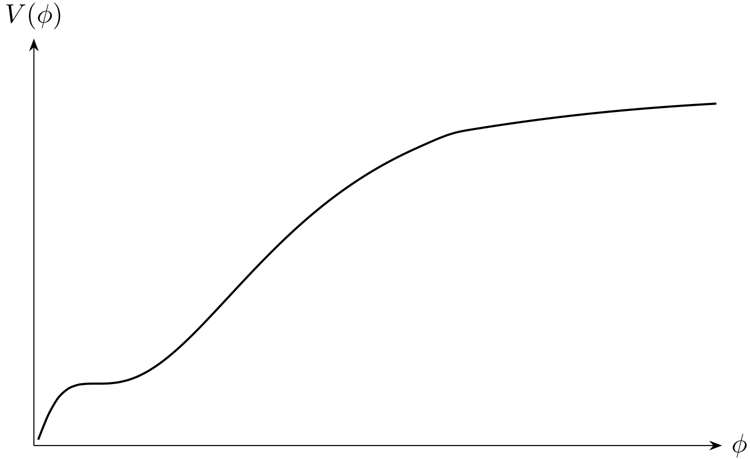
Stochastic inflation

- Includes non-linear effects
- Numerical method: even more non-linearities

Model of inflation fits CMB



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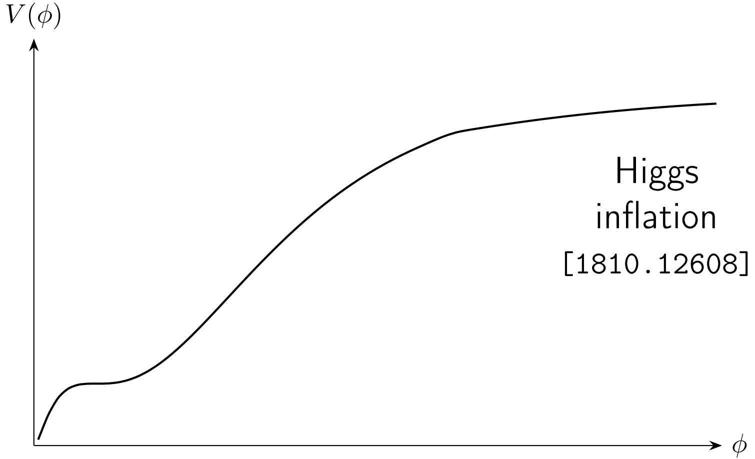

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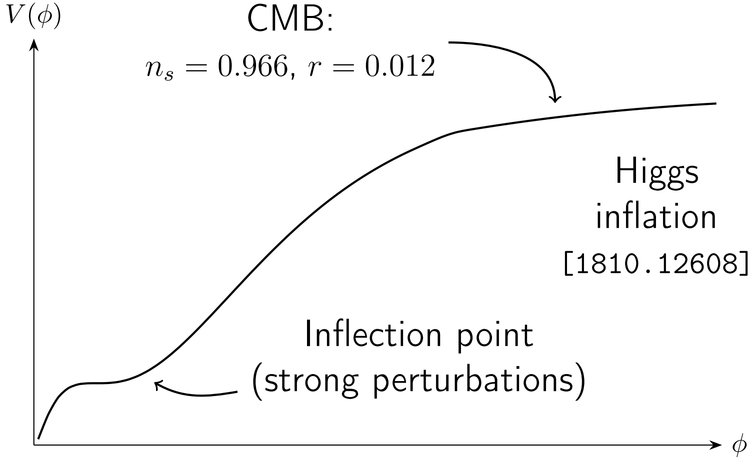

Perturbations depend on scale

Origin of perturbations: fluctuations of quantum vacuum

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Space expands and perturbations get stretched

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Strong perturbations from ultra-slow-roll inflation

Must go beyond linear perturbations

Coarse-grain perturbations over super-Hubble scales

Gradient expansion: to leading order, coarse-grained perturbations follow locally (non-linear) FLRW equations `[Class.Q.Grav.9,1943(1992)]`

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ΔN formalism: from FLRW variables to perturbation variables [astro-ph/9507001]

- Change in e-folds of expansion $\Delta N =$ curvature perturbation $\mathcal{R}$

Stretching perturbations give stochastic kicks

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Stochastic evolution of local coarse-grained field
`[Lect.Notes Phys.246,107(1986)]`

PBHs form from strong perturbations

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[1309.4201, 1405.7023, 2011.03014]

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BH mass = all the mass inside one Hubble radius when the scale re-enters

ALGORITHM

Track numerically evolution of coarse-grained field $\bar{\phi}$ and linear perturbations $\delta\phi$

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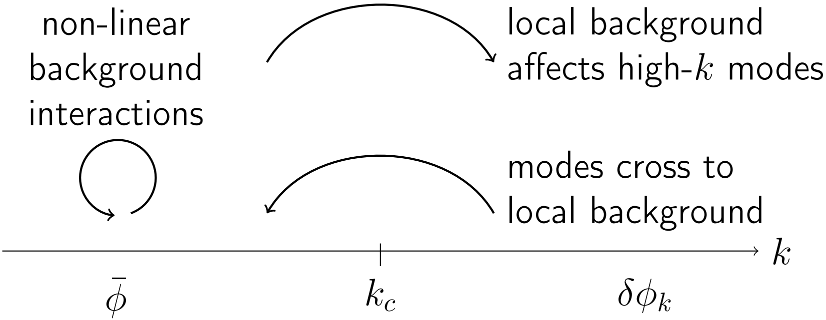
Stochastic evolution with backreaction

Non-linear interactions included



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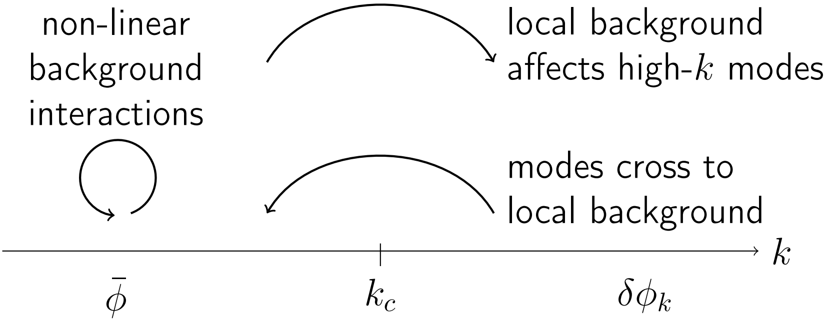

Compare to simpler approach with noise $\sim \frac{H^2}{2\pi^2}$

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Repeat 10^{11} times, collect statistics

Want tiny initial PBH fraction

Scale $M_{\rm PBH} = 10^{-14} M_\odot$, $k_{\rm PBH} = 10^{13}$ Mpc^{-1} chosen so that USR ends when $k_{\rm PBH}$ gets coarse-grained

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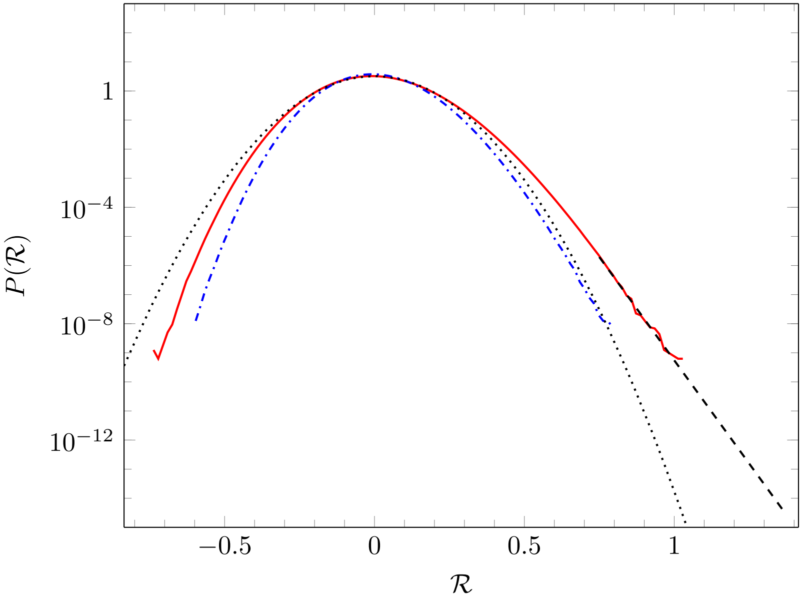
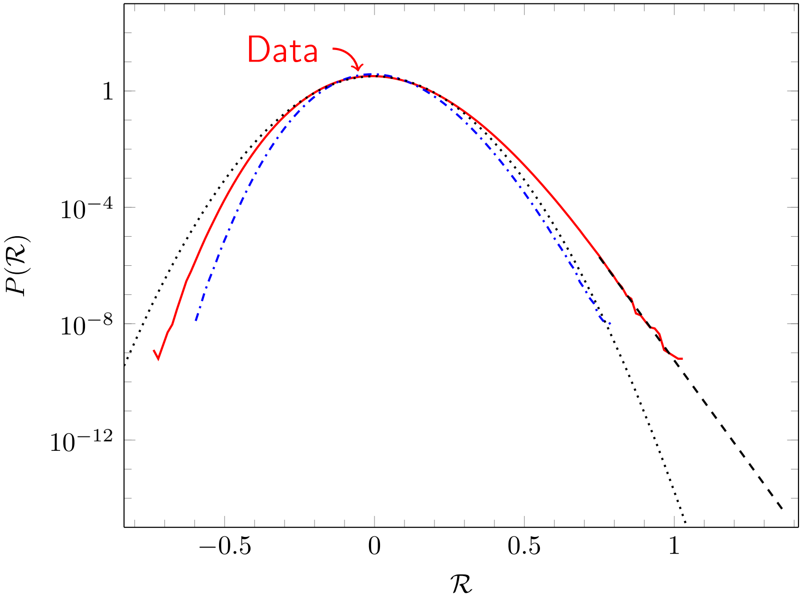
Gaussian statistics:

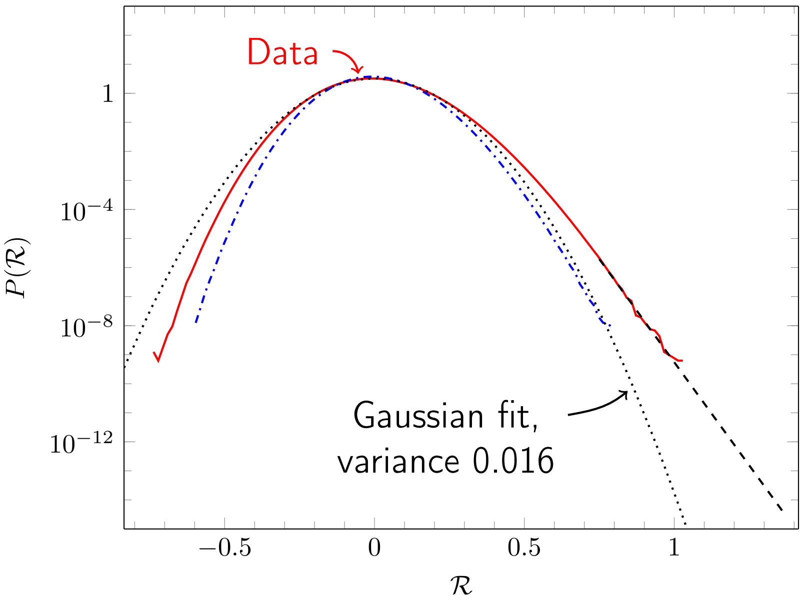
$$\sigma_{\mathcal{R}}^2 = \int^{k_{\rm PBH}} \mathrm{d}(\ln k) \mathcal{P}_{\mathcal{R}}(k)$$

$$\beta = 2 \int_{\mathcal{R}_c}^{\infty} \mathrm{d}\mathcal{R} \frac{1}{\sqrt{2\pi}\sigma_{\mathcal{R}}} e^{-\frac{\mathcal{R}^2}{2\sigma_{\mathcal{R}}^2}} \approx \frac{\sqrt{2}\sigma_{\mathcal{R}}}{\sqrt{\pi}\mathcal{R}_c} e^{-\frac{\mathcal{R}_c^2}{2\sigma_{\mathcal{R}}^2}}$$

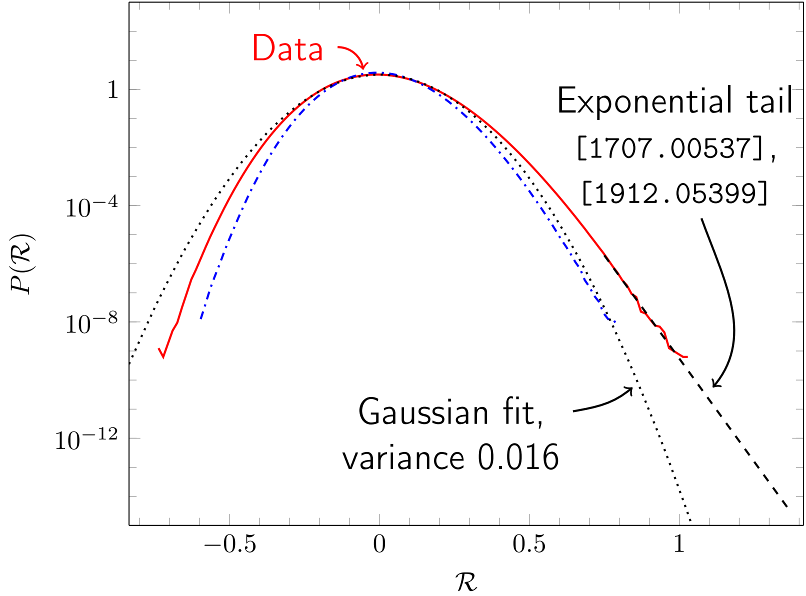


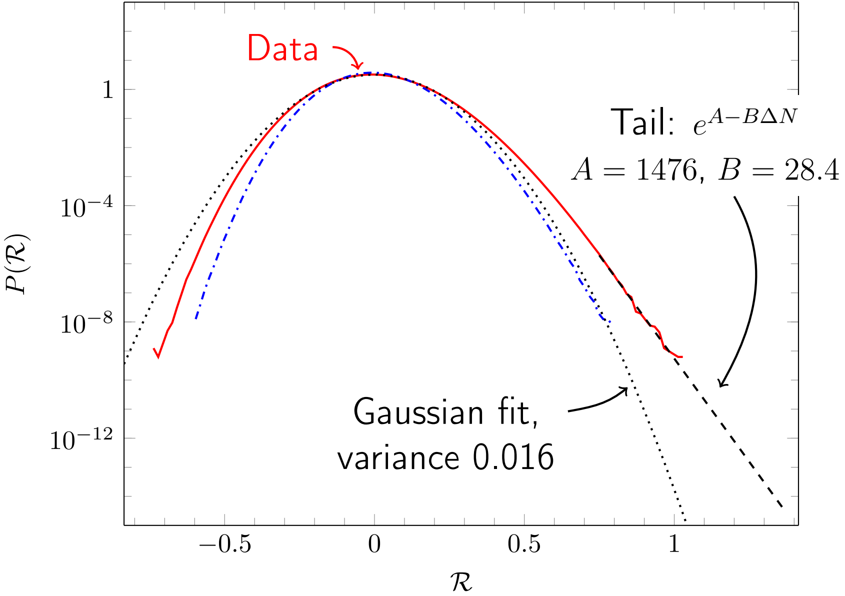
1
10^{-4}
10^{-8}
10^{-12}
$P(\mathcal{R})$
−0.5
0
0.5
1
$\mathcal{R}$


Data
1
10−4
10−8
10−12
P(R)
−0.5
0
0.5
1
R

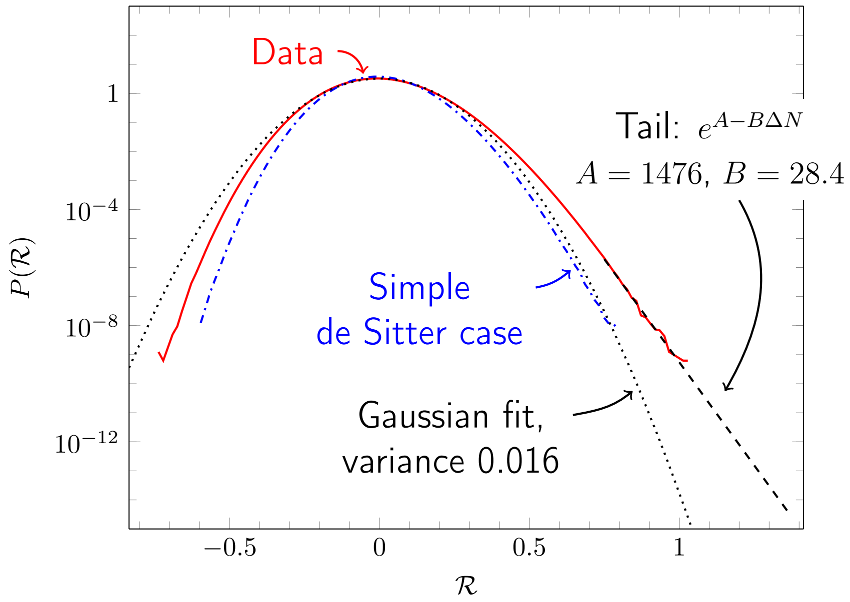


Data
Gaussian fit,
variance 0.016
$P(\mathcal{R})$
$\mathcal{R}$
1
10^{-4}
10^{-8}
10^{-12}
-0.5
0
0.5
1

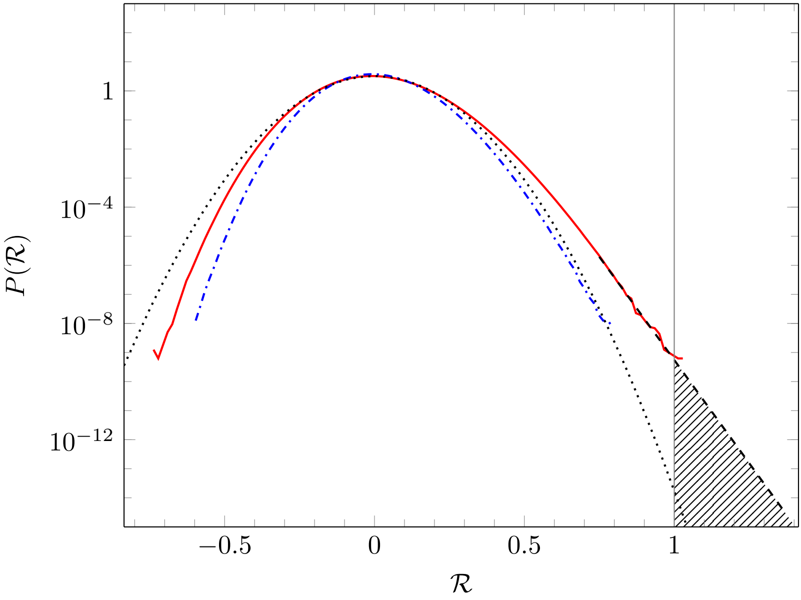

Data
Exponential tail
[1707.00537],
[1912.05399]
Gaussian fit,
variance 0.016
1
10^{-4}
10^{-8}
10^{-12}
$P(\mathcal{R})$
−0.5
0
0.5
1
$\mathcal{R}$



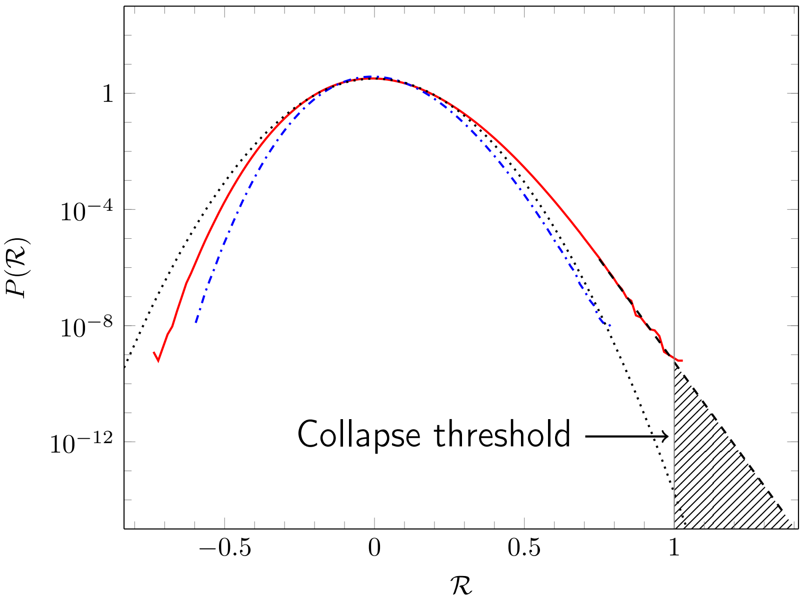
Data
Tail: $e^{A-B\Delta N}$
$A = 1476$, $B = 28.4$
Gaussian fit,
variance 0.016
1
10^{-4}
10^{-8}
10^{-12}
$P(\mathcal{R})$
−0.5
0
0.5
1
$\mathcal{R}$



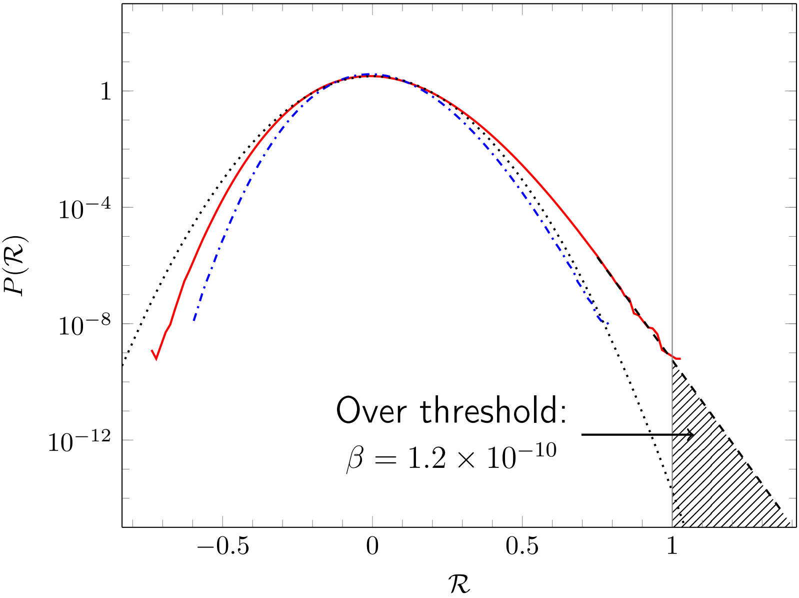
Data
Tail: $e^{A-B\Delta N}$
$A = 1476,\ B = 28.4$
Simple
de Sitter case
Gaussian fit,
variance 0.016
$P(\mathcal{R})$
1
10^{-4}
10^{-8}
10^{-12}
-0.5
0
0.5
1
$\mathcal{R}$



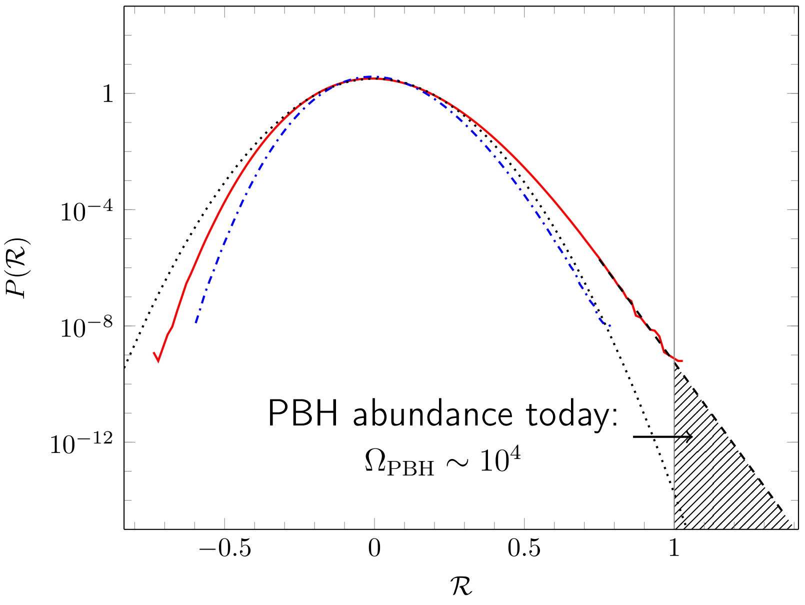
1
10^{-4}
10^{-8}
10^{-12}
$P(\mathcal{R})$
-0.5
0
0.5
1
$\mathcal{R}$



1
10^{-4}
10^{-8}
10^{-12}
$P(\mathcal{R})$
Collapse threshold
-0.5
0
0.5
1
$\mathcal{R}$



1
10^{-4}
10^{-8}
10^{-12}
$P(\mathcal{R})$
−0.5
0
0.5
1
$\mathcal{R}$
Over threshold:
$\beta = 1.2 \times 10^{-10}$



1
10^{-4}
10^{-8}
10^{-12}
$P(\mathcal{R})$
-0.5
0
0.5
1
$\mathcal{R}$
PBH abundance today:
$\Omega_{\rm PBH} \sim 10^4$

True abundance much higher than Gaussian estimate

Numerics: exponential tail, with

$$\beta = 1.2 \times 10^{-10}\,, \quad \Omega_{\rm PBH} = 5.4 \times 10^{4}$$

Larger than Gaussian result by factor 10^5!

Future directions

More statistics

More models

Full mass spectrum

Correlations between different scales

Conclusions

Stochastic inflation captures non-linearities of cosmological perturbations

Crucial for PBH formation

Introduced a numerical recipe to calculate these in a general single-field model

Thank you!

[2012.06551]