Stochastic inflation and primordial black holes

Tallinn, 3 March 2022

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Based on 2012.06551, 2110.10684, 2111.07437, in collaboration with D. Figueroa, S. Raatikainen, S. Räsänen

Concepts

Cosmic inflation

- Accelerating expansion of space in the early universe

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Cosmological perturbations

- Cosmic microwave background, ...

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Primordial black holes

- Dark matter candidate

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- Includes non-linear effects

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Stochastic inflation

- Includes non-linear effects
- Numerical method: even more non-linearities

Part I:

Overview

Cosmic inflation

Hypothetical era in the early universe with accelerating expansion: $\ddot{a}(t) > 0$

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Explains origin of cosmological perturbations

Cosmic inflation with a scalar field

$\ddot{a}(t) > 0$ accomplished by scalar field matter

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2}M_P^2 R - \frac{1}{2}(\partial_\mu\phi)^2 - V(\phi)\right)$$

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$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$3H^2 M_P^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi)\,, \quad H \equiv \frac{\dot{a}}{a}$$

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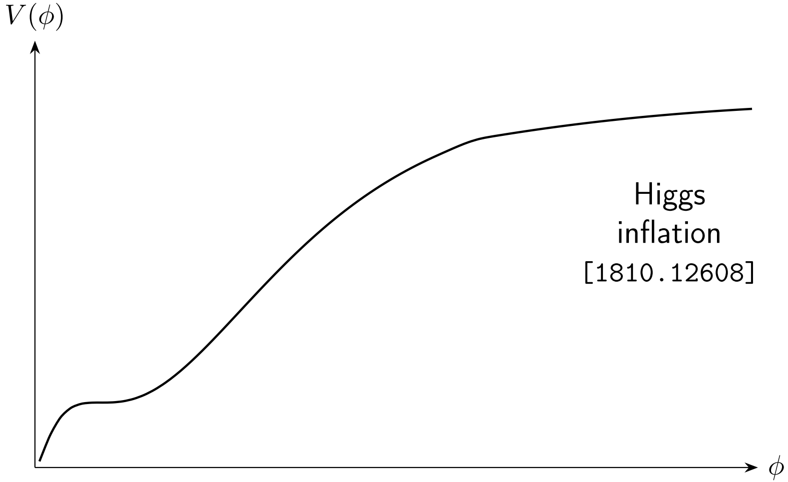
Inflation happens when $V(\phi)$ dominates over $\dot{\phi}^2$

Our example model is typical



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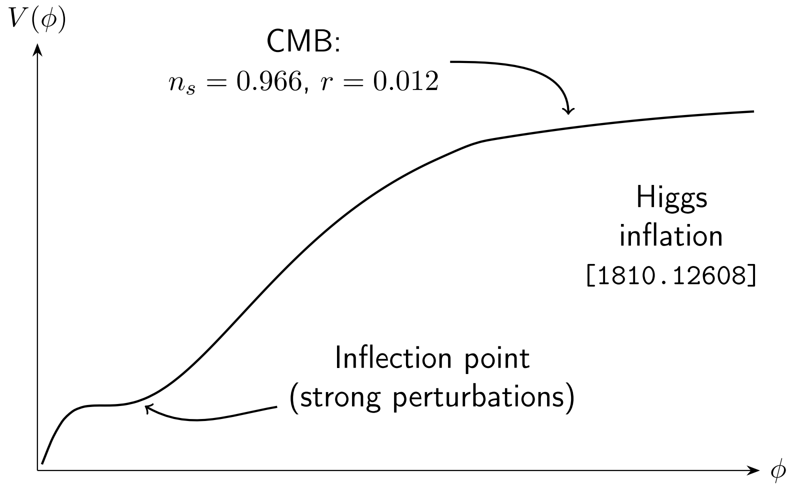

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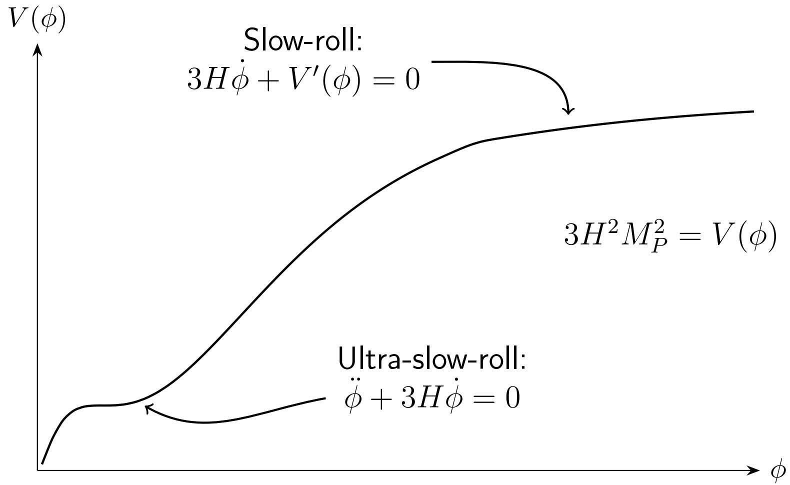

Perturbations depend on scale

Origin of perturbations: fluctuations of quantum vacuum

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Space expands and perturbations get stretched

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Strong perturbations from ultra-slow-roll inflation

Linear perturbation theory

Expand to linear order:

$$\delta\ddot{\phi}_{\vec{k}} + 3H\delta\dot{\phi}_{\vec{k}} + V''(\phi)\delta\phi_{\vec{k}} = 0$$

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Comoving curvature perturbation and its power spectrum:

$$\mathcal{R}_{\vec{k}} = \frac{\delta\phi_{\vec{k}} H}{\dot{\phi}}\,, \qquad \mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}|\mathcal{R}_{\vec{k}}|^2$$

CMB observables in slow-roll

$$\mathcal{P}_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_*}\right)^{n_s-1}, \qquad A_s = \frac{V}{24\pi^2 \epsilon_V}\,,$$

$$n_s = 1 - 6\epsilon_V + 2\eta_V\,, \qquad r = 16\epsilon_V\,,$$

$$\epsilon_V = \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2, \quad \eta_V = M_P^2 \frac{V''}{V}\,.$$

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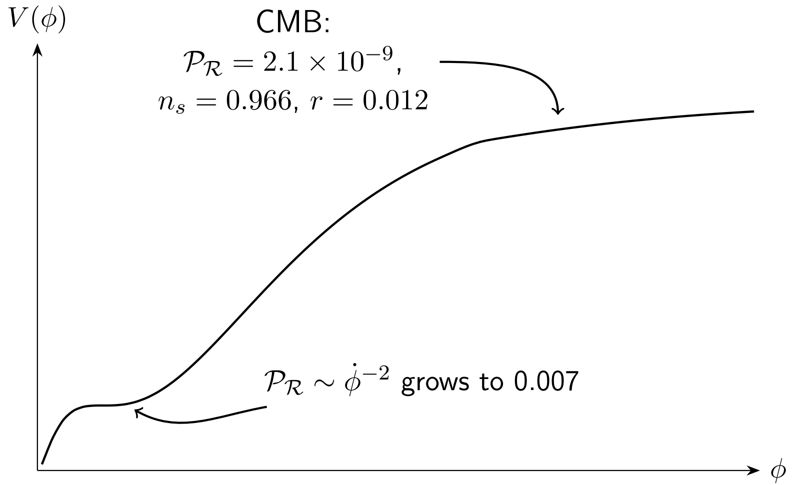
Observations (Planck):

$$k_* = 0.05\text{Mpc}^{-1}, \quad A_s \approx 2.1 \times 10^{-9},$$

$$n_s \approx 0.96, \qquad r \lesssim 0.08$$

Our example model



Must go beyond linear perturbations

Coarse-grain perturbations over super-Hubble scales

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Gradient expansion: to leading order, coarse-grained perturbations follow locally (non-linear) FLRW equations
[Class.Q.Grav.9,1943(1992)]

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Gradient expansion: to leading order, coarse-grained perturbations follow locally (non-linear) FLRW equations [Class.Q.Grav.9,1943(1992)]

ΔN formalism: from FLRW variables to perturbation variables [astro-ph/9507001]

- Change in e-folds of expansion $\Delta N = \Delta \ln a =$ curvature perturbation $\mathcal{R}$

Stretching perturbations give stochastic kicks

When perturbations of a certain scale stretch to the coarse-graining scale, they get coarse-grained

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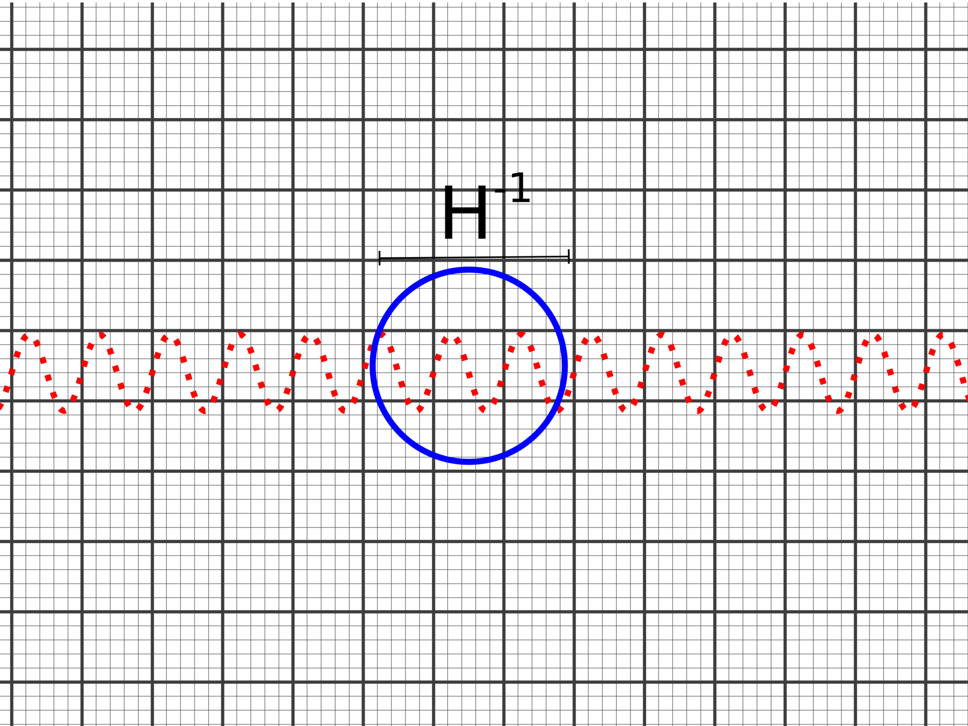
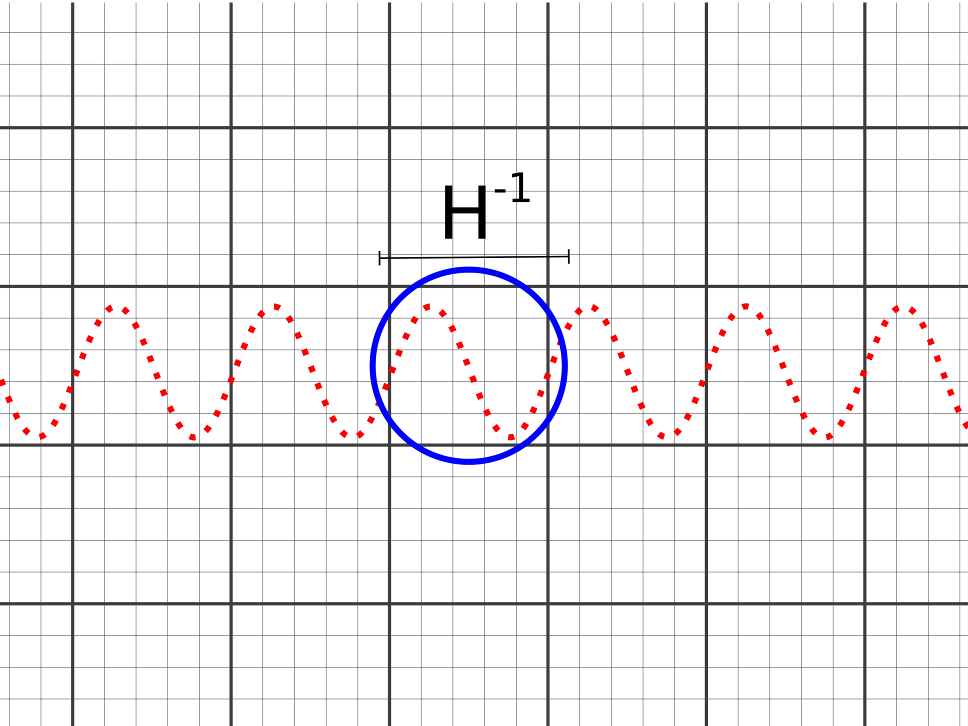
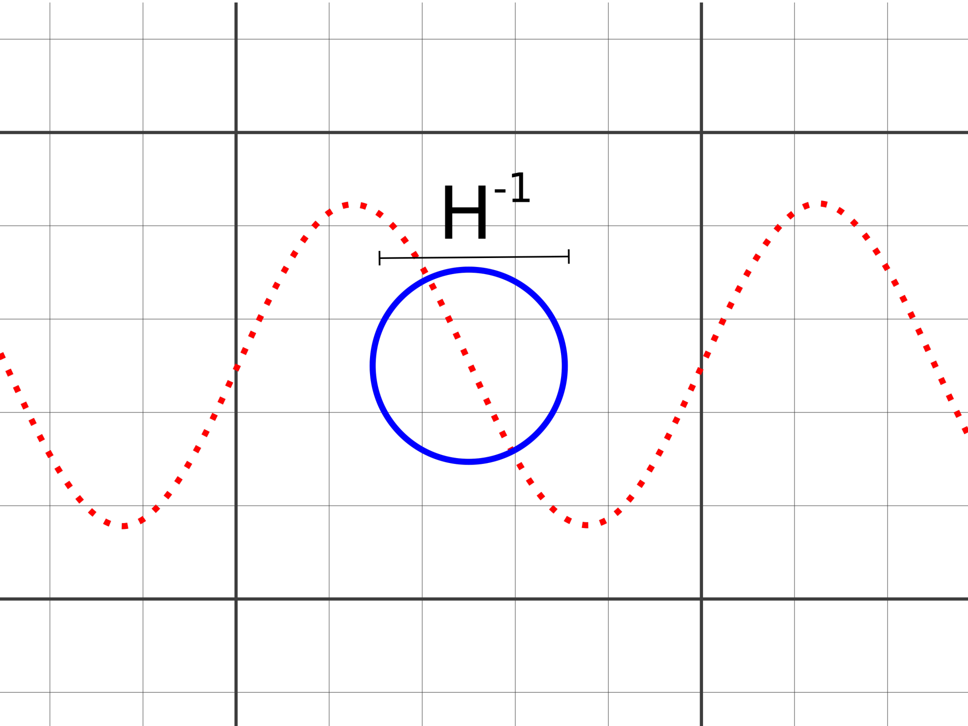
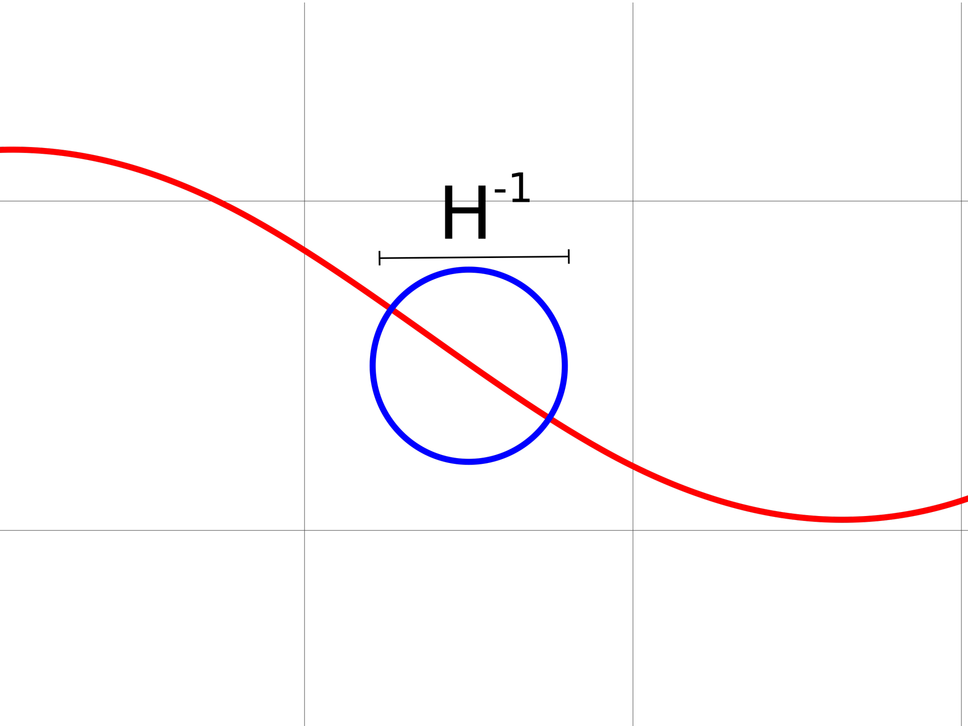
Stochastic evolution of local coarse-grained field
`[Lect.Notes Phys.246,107(1986)]`



H-1


H^{-1}


H^{-1}


H-1

PBHs form from strong perturbations

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Perturbation collapses to black hole if it exceeds threshold [1309.4201, 1405.7023, 2011.03014]

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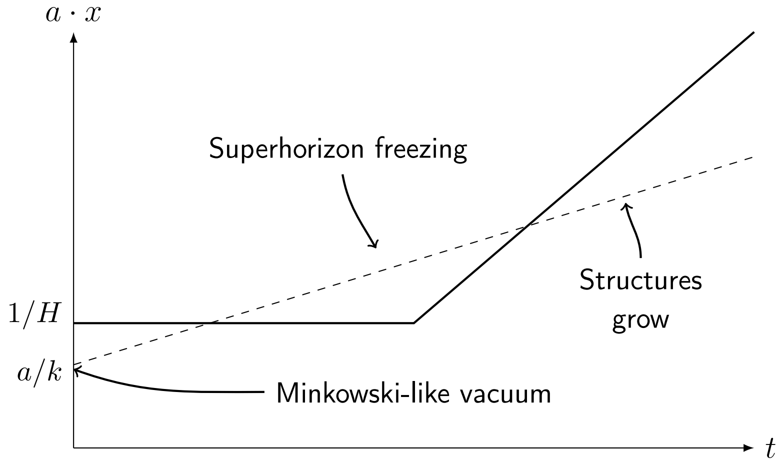
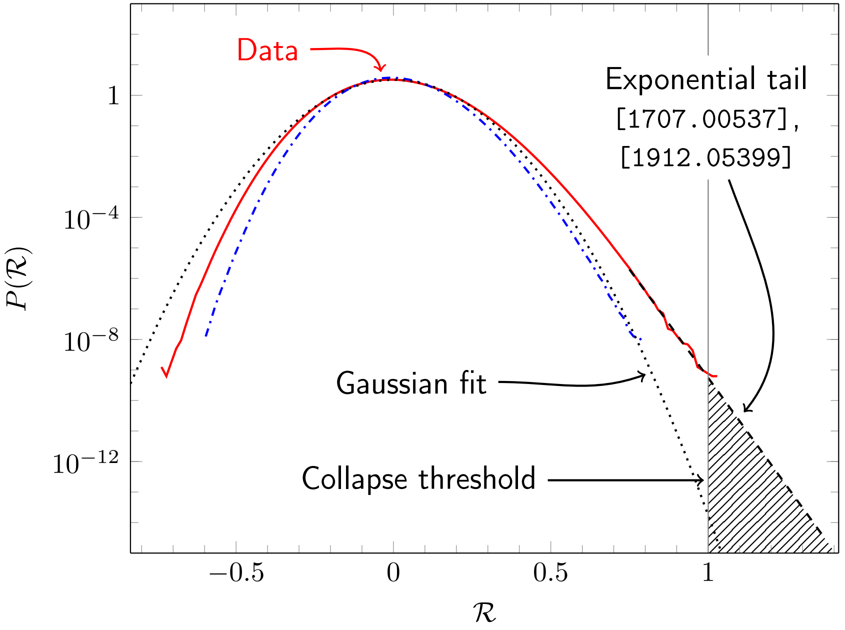
During radiation domination, perturbations re-enter Hubble radius

Perturbation collapses to black hole if it exceeds threshold [1309.4201, 1405.7023, 2011.03014]

BH mass = all the mass inside one Hubble radius when the scale re-enters

Evolution of length scales




Data
Exponential tail
[1707.00537],
[1912.05399]
Gaussian fit
Collapse threshold
$P(\mathcal{R})$
$\mathcal{R}$
1
10^{-4}
10^{-8}
10^{-12}
−0.5
0
0.5
1

Part II:

Technical details

Dividing the field

Divide inflaton field ϕ into coarse-grained and short wavelength perturbations:

$$\phi = \underbrace{\int_{k<k_\mathrm{c}} \frac{\mathrm{d}^3k}{(2\pi)^{3/2}}\,\phi_{\vec{k}}\,e^{-i\vec{k}\cdot\vec{x}}}_{\bar{\phi}} + \underbrace{\int_{k>k_\mathrm{c}} \frac{\mathrm{d}^3k}{(2\pi)^{3/2}}\,\phi_{\vec{k}}\,e^{-i\vec{k}\cdot\vec{x}}}_{\delta\phi}$$

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Define coarse-grained field momentum:

$$\bar{\pi} = \int_{k<k_\mathrm{c}} \frac{\mathrm{d}^3 k}{(2\pi)^{3/2}}\, \frac{\partial}{\partial N} \phi_{\vec{k}}\, e^{-i\vec{k}\cdot\vec{x}}$$

Coarse-graining induces noise

Time derivatives:

$$\bar{\phi}' = \bar{\pi} + \xi_\phi$$

$$\bar{\pi}' = \int_{k<k_\mathrm{c}} \frac{\mathrm{d}^3 k}{(2\pi)^{3/2}} \, \frac{\partial^2}{\partial N^2} \phi_{\vec{k}} \, e^{-i\vec{k}\cdot\vec{x}} + \xi_\pi$$

ξ_ϕ, ξ_π are noise from drifting Fourier-modes: random due to quantum initial conditions, with $\left\langle \xi_\phi^2 \right\rangle \sim |\phi_{k_c}|^2$, $\langle \xi_\pi^2 \rangle \sim |\pi_{k_c}|^2$

Field equations become stochastic

Full scalar field equation:

$\partial^\mu \partial_\mu \phi - V'(\phi) = 0$

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Full scalar field equation:

$$\phi'' + \left(3 + \frac{H'}{H}\right)\phi' - \frac{1}{a^2 H^2}\nabla^2\phi + \frac{V'(\phi)}{H^2} = 0$$

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Full scalar field equation:

$$\int \frac{\mathrm{d}^3 k}{(2\pi)^{3/2}}\, \frac{\partial^2}{\partial N^2} \phi_{\vec{k}}\, e^{-i\vec{k}\cdot\vec{x}} + \left(3 + \frac{H'}{H}\right)(\bar{\pi} + \delta\pi)$$

$$-\frac{1}{a^2 H^2} \nabla^2 \bar{\phi} - \frac{1}{a^2 H^2} \nabla^2 \delta\phi$$

$$+\frac{1}{H^2}\left(V'(\bar{\phi}) + \frac{1}{2} V''(\bar{\phi})\delta\phi + \frac{1}{6} V'''(\bar{\phi})\delta\phi^2 + \dots\right)$$

$$= 0$$

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$$-\frac{1}{a^2H^2}\nabla^2\bar{\phi}\;(\to 0) - \frac{1}{a^2H^2}\nabla^2\delta\phi$$

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$$\delta\phi''_{\vec{k}} + \left(3 + \frac{H'}{H}\right)\delta\phi'_{\vec{k}} + \left(\frac{k^2}{a^2H^2} + \frac{1}{H^2}V''(\bar{\phi})\right)\delta\phi_{\vec{k}} = 0$$

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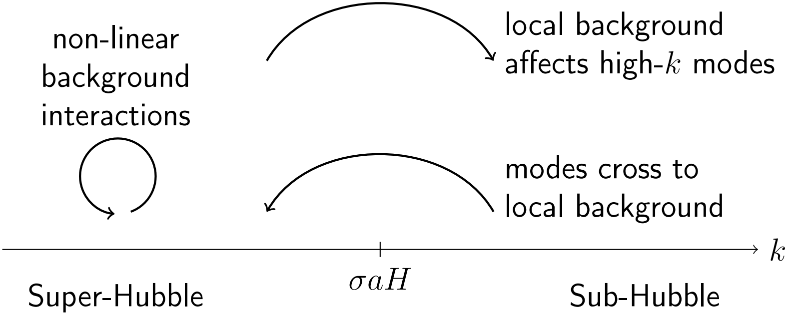
$$3H^2 = \tfrac{1}{2}\bar{\pi}^2 + V(\bar{\phi})$$

Non-linear interactions included



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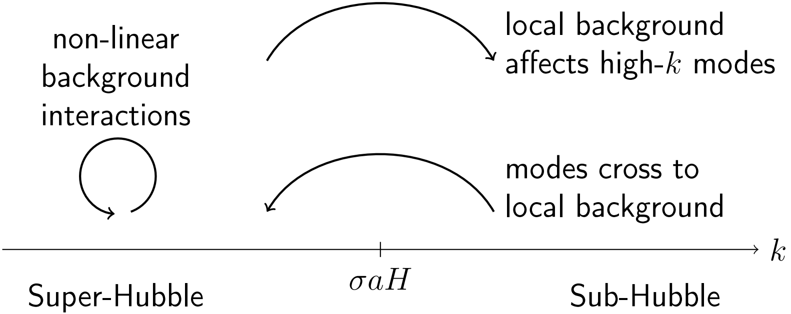

Compare to simpler approach with noise $\sim \frac{H^2}{2\pi^2}$

Perturbations start in Bunch-Davies vacuum

Perturbation initial conditions are

$$\delta\phi_{\vec{k}} = \frac{1}{a\sqrt{2k}}\,, \qquad \delta\phi'_{\vec{k}} = -\big(1 + i\frac{k}{aH}\big)\delta\phi_{\vec{k}}$$

We follow modes from deep within the Hubble radius to coarse-graining scale to get the kicks

Discrete time steps give finite kicks

Free quantum scalar field: Gaussian statistics, white noise,

$$\left\langle \xi_\phi^2 \right\rangle = \left\langle (\Delta\bar{\phi})^2 \right\rangle = \mathrm{d}N \frac{k^3}{2\pi^2}\left(1 + \frac{H'}{H}\right) \left|\delta\phi_{\vec{k}}\right|^2$$

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Squeezed state: ξ_ϕ and ξ_π are highly correlated, so that

$$\Delta\bar{\pi} = \frac{\delta\phi'_{\vec{k}}}{\delta\phi_{\vec{k}}}\Delta\bar{\phi}$$

Kicks are turned off when target scale reached

We are interested in PBHs with a specific mass $M_{\rm PBH}$, corresponding to a specific scale $k_{\rm PBH}$

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After this scale gets coarse-grained, no more kicks

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- Coarse-grained patch has correct size for PBH formation
- Shorter wavelengths don't contribute: they are 'smoothed over'

ALGORITHM

Track numerically evolution of coarse-grained field $\bar{\phi}$ and linear perturbations $\delta\phi$

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Continue (without kicks) to constant-ϕ hypersurface, record N

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Continue (without kicks) to constant-ϕ hypersurface, record N

Repeat 10^{11} times, collect statistics

Part III:

Numerical results

Want tiny initial PBH fraction

PBH fraction today:

$$\Omega_{\mathsf{PBH}} \approx 9 \times 10^7 \gamma^{\frac{1}{2}} \beta \left(\frac{M}{M_\odot} \right)^{-\frac{1}{2}} \sim 0.3$$

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Our example: asteroid mass PBHs,
$M_{\rm PBH} = 10^{-14} M_\odot$, $k_{\rm PBH} = 10^{13}$ Mpc^{-1}
(USR ends when $k_{\rm PBH}$ gets coarse-grained)

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Need initial fraction $\beta \sim 10^{-16}$

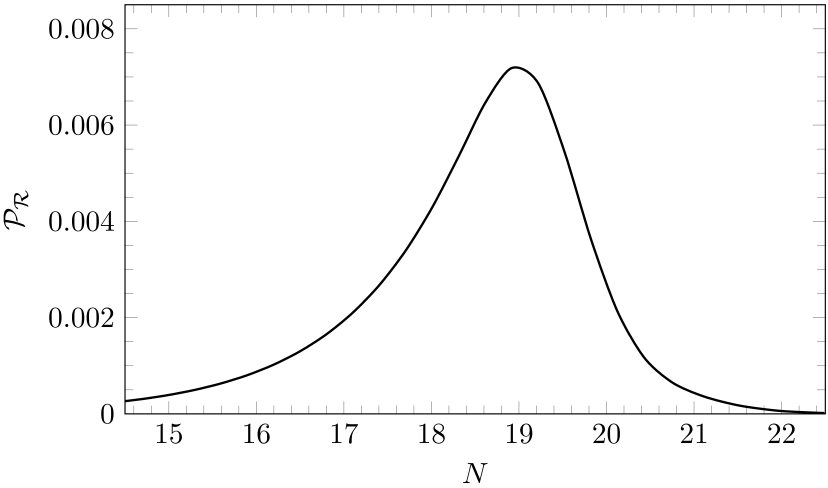
Model fitted by Gaussian approximation

With Gaussian statistics:

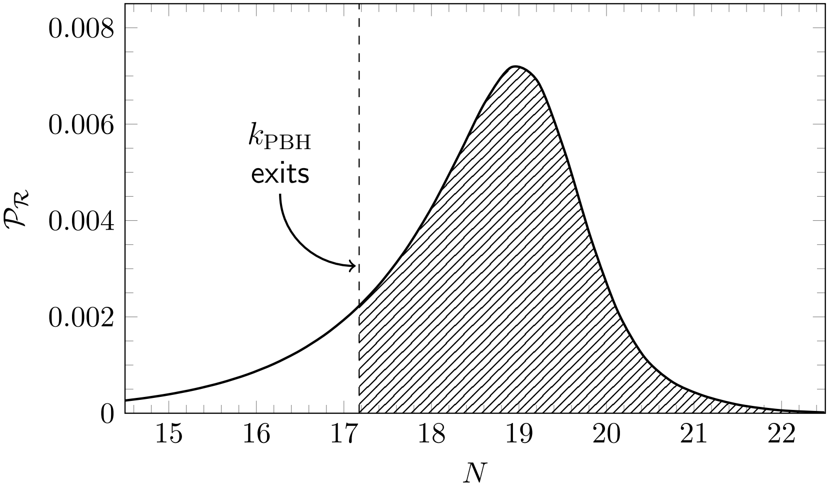
$$\sigma_{\mathcal{R}}^2 \equiv \int^{k_{\rm PBH}} \mathrm{d}(\ln k) \mathcal{P}_{\mathcal{R}}(k)$$

$$\beta = 2 \int_{\mathcal{R}_c}^{\infty} \mathrm{d}\mathcal{R} \frac{1}{\sqrt{2\pi}\sigma_{\mathcal{R}}} e^{-\frac{\mathcal{R}^2}{2\sigma_{\mathcal{R}}^2}} \approx \frac{\sqrt{2}\sigma_{\mathcal{R}}}{\sqrt{\pi}\mathcal{R}_c} e^{-\frac{\mathcal{R}_c^2}{2\sigma_{\mathcal{R}}^2}}$$

Model fitted by Gaussian approximation



Model fitted by Gaussian approximation



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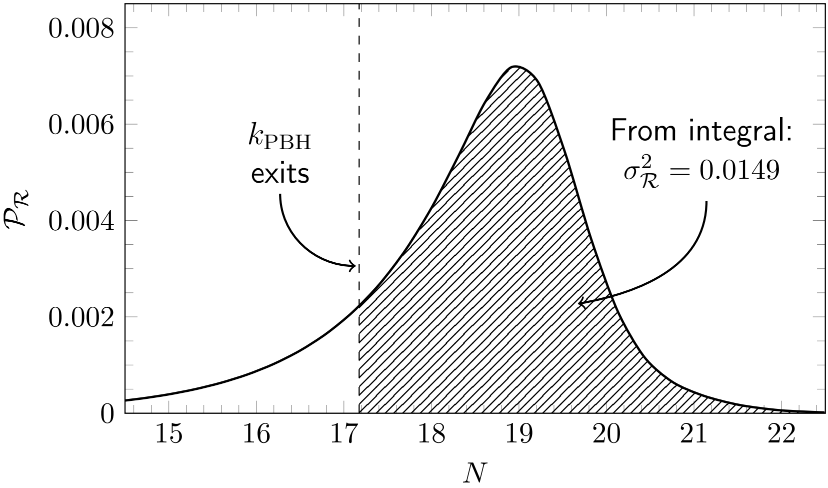

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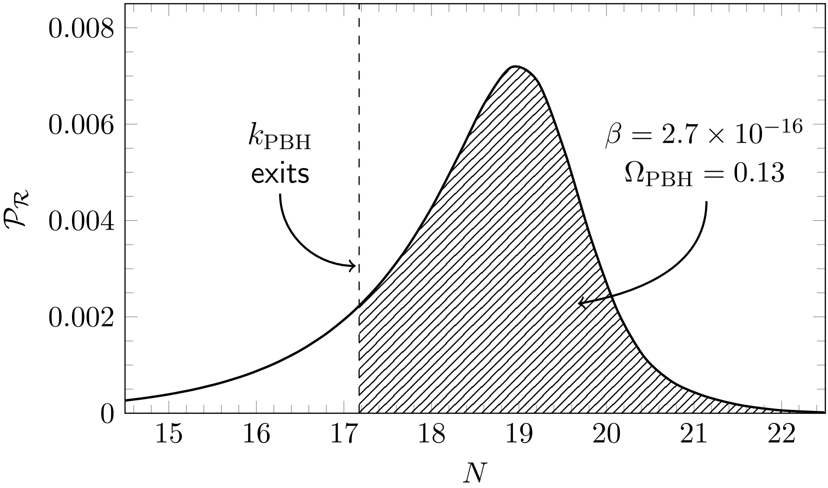
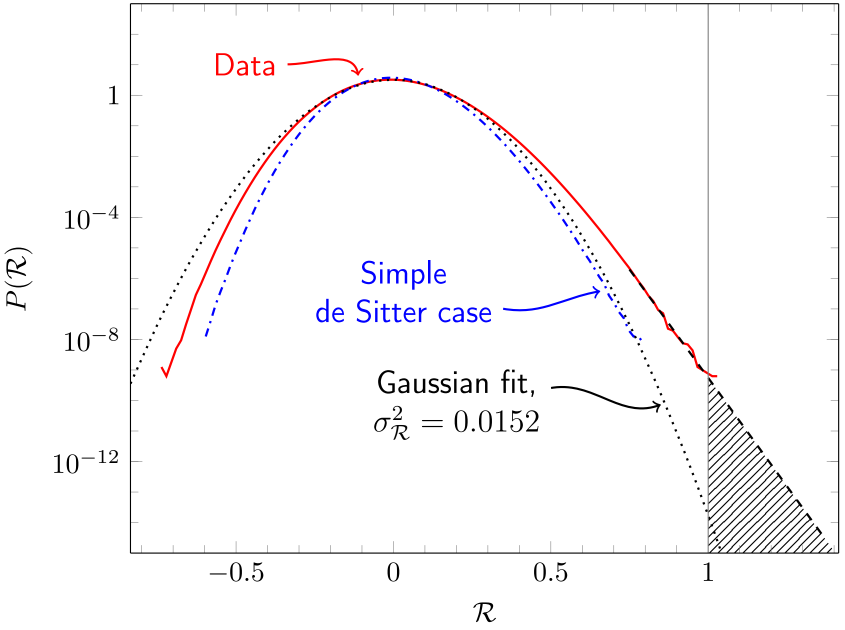
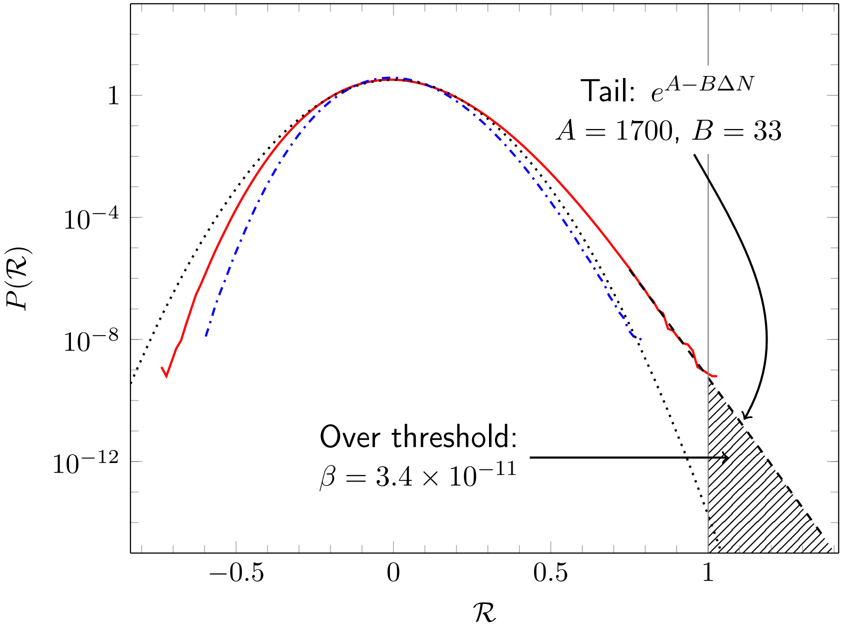
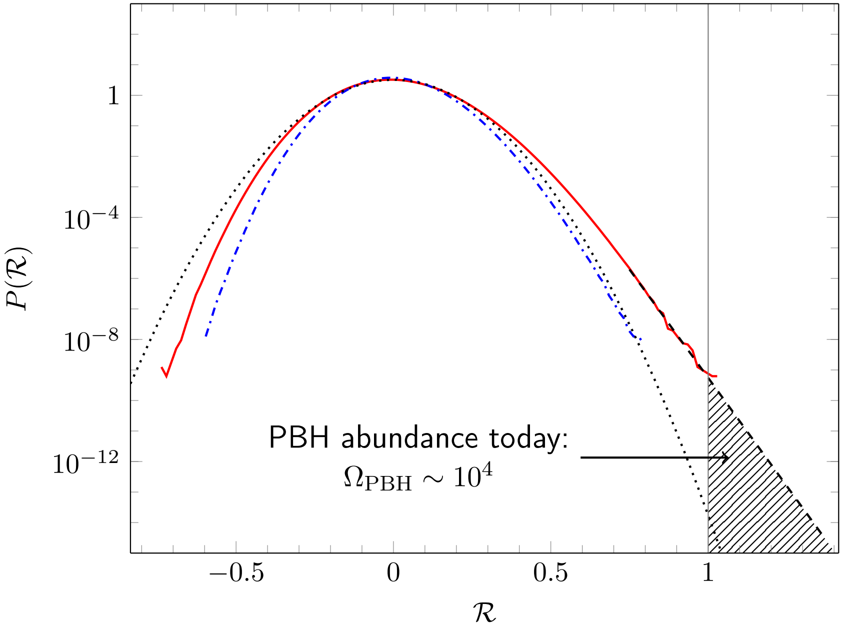



Data
1
10^{-4}
10^{-8}
10^{-12}
$P(\mathcal{R})$
Simple
de Sitter case
Gaussian fit,
$\sigma_{\mathcal{R}}^2 = 0.0152$
-0.5
0
0.5
1
$\mathcal{R}$



Tail: $e^{A-B\Delta N}$
$A = 1700,\ B = 33$
Over threshold:
$\beta = 3.4 \times 10^{-11}$
1
10^{-4}
10^{-8}
10^{-12}
$P(\mathcal{R})$
−0.5
0
0.5
1
$\mathcal{R}$



PBH abundance today:
$\Omega_{\rm PBH} \sim 10^4$
1
10^{-4}
10^{-8}
10^{-12}
$P(\mathcal{R})$
−0.5
0
0.5
1
$\mathcal{R}$

...true abundance much higher

Numerics: exponential tail, with

$$\beta = 3.4 \times 10^{-11}\,, \quad \Omega_{\rm PBH} = 1.6 \times 10^{4}$$

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Larger than Gaussian result by factor 10^5!

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$$\beta = 3.4 \times 10^{-11}\,, \quad \Omega_{\rm PBH} = 1.6 \times 10^{4}$$

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Other sources of error: uncertainty in $\mathcal{R}_c$, window functions, different Gaussian computation schemes, ...

Alternate setups

Performed simulations in three ways:

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1. Full simulation with modes and backreaction

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1. Full simulation with modes and backreaction
2. Modes from unperturbed background, no backreaction
3. No modes; noise $\sim H^2$

Results: 2 is identical to 1; 3 is not. Backreaction on modes not important; mode evolution is!

Other scenarios

We also studied other similar potentials, tuned to produce PBHs of different masses:

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- Solar mass: $M = 4.7 M_{\odot}$,
  $\Omega_{\text{PBH}}^{\text{Gauss}} = 0.17$, $\Omega_{\text{PBH}}^{\text{data}} = 1.6$, $\Omega_{\text{PBH}}^{\text{de Sitter}} = 125$

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- Solar mass: $M = 4.7 M_\odot$,
  $\Omega_{\text{PBH}}^{\text{Gauss}} = 0.17$, $\Omega_{\text{PBH}}^{\text{data}} = 1.6$, $\Omega_{\text{PBH}}^{\text{de Sitter}} = 125$
- Galaxy seeds: $M = 1.8 \times 10^3 M_\odot$,
  $\Omega_{\text{PBH}}^{\text{Gauss}} = 1.4 \times 10^{-5}$, $\Omega_{\text{PBH}}^{\text{data}} = 0.05$, $\Omega_{\text{PBH}}^{\text{de Sitter}} = 17$

Other scenarios

We also studied other similar potentials, tuned to produce PBHs of different masses:

- Solar mass: $M = 4.7 M_\odot$,
  $\Omega_{\text{PBH}}^{\text{Gauss}} = 0.17$, $\Omega_{\text{PBH}}^{\text{data}} = 1.6$, $\Omega_{\text{PBH}}^{\text{de Sitter}} = 125$
- Galaxy seeds: $M = 1.8 \times 10^3 M_\odot$,
  $\Omega_{\text{PBH}}^{\text{Gauss}} = 1.4 \times 10^{-5}$, $\Omega_{\text{PBH}}^{\text{data}} = 0.05$, $\Omega_{\text{PBH}}^{\text{de Sitter}} = 17$
- Planck mass relics: $M = 1.4 \times 10^3$ kg,
  $\Omega_{\text{PBH}}^{\text{Gauss}} = 0.11$, $\Omega_{\text{PBH}}^{\text{data}} = 2.4 \times 10^7$, $\Omega_{\text{PBH}}^{\text{de Sitter}} = 5 \times 10^{-24}$

Future directions

Reducing numerical load

Correlations between different scales

PBH statistics from exponential tail

Conclusions

Inflation produces cosmological perturbations; strongest collapse to black holes

Non-Gaussian tail of probablity distribution important for black hole statistics

Stochastic inflation allows us to probe this

Numerical simulations improve accuracy; mode evolution is important, backreaction not

Thank you!

What about σ?

Coarse-graining parameter $\sigma < 1$ is a free parameter

- Results may depend on it

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Coarse-graining parameter $\sigma < 1$ is a free parameter

- Results may depend on it

Want to make a physically well-motivated choice

- Want a lot of non-linear interactions: large σ
- Want kicks to be classical: small σ

Demanding high squeezing sets σ

Classicality measured by squeezing of quantum state

- Squeezed state: phase space probability distribution classial
- Also, ξ_ϕ and ξ_π correlated

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$$\cosh(2r_k) = a^3\big(\tfrac{k}{a}|\delta\phi_k|^2 + \tfrac{a}{k}H^2|\delta\phi'_k|^2\big)$$

Our choice: $\sigma = 0.01$ ensures $\cosh(2r_k) > 100$ for all modes when they exit k_c

What about gauge issues?

$\delta\phi$ and thus kicks solved in spatially flat gauge

- Easy to solve

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To have no kicks in scale factor, need uniform-N gauge

Tests and theory: no significant difference [1905.06300]

Model details

$V = \frac{\lambda(h)}{4} F(h)^4$

$F(h) = \frac{Ah}{\sqrt{1+B\xi(h-C)^2}}$, $\qquad \frac{\mathrm{d}h}{\mathrm{d}\chi} = \frac{1+\xi h^2}{\sqrt{1+\xi h^2+6\xi^2 h^2}}$

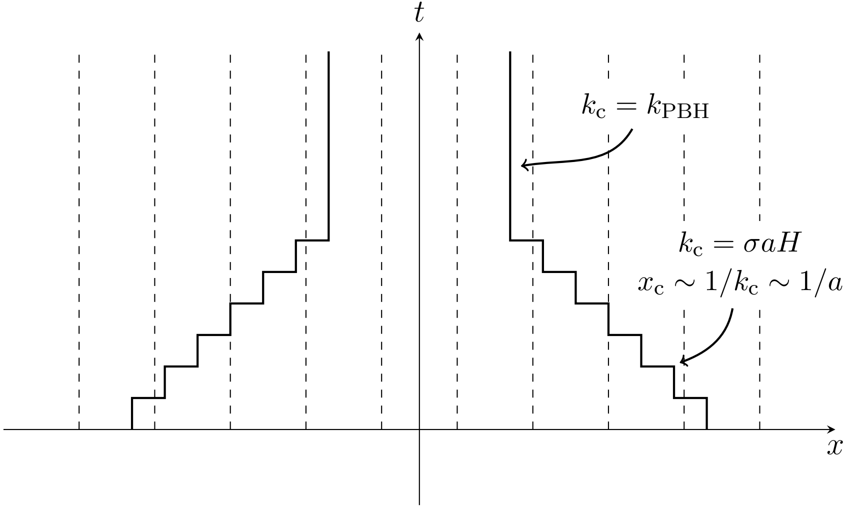
$\xi = 38.8$

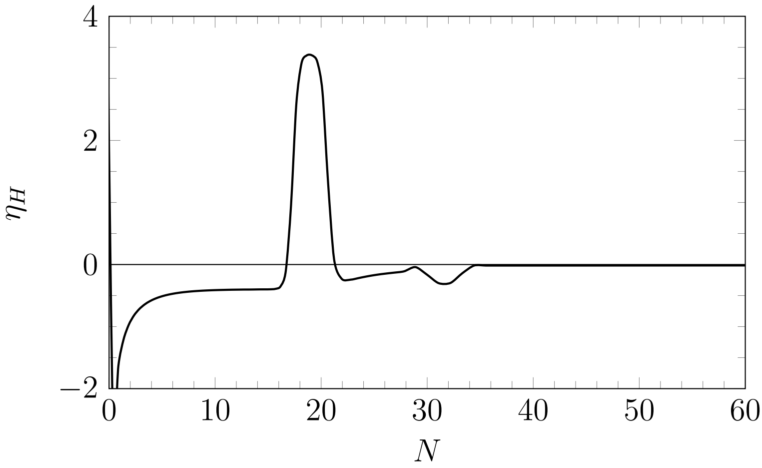
$n_s = 0.966$, $r = 0.012$, $A_s = 2.1 \times 10^{-9}$

USR between 17.2 and 20.8 e-folds

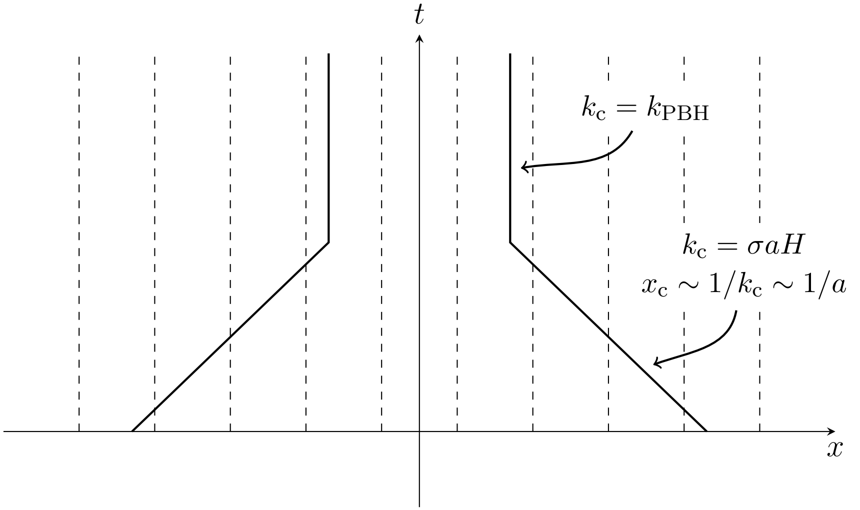
[1810.12608]

Evolution of patch size




4
2
0
−2
η_H
0
10
20
30
40
50
60
N

Evolution of patch size



Algorithm 1: Evolution for each run

Set initial values for N, $\bar{\phi}$, $\bar{\pi}$. Set $k_{\rm next} = k_*$. Set current kick coefficient to zero.

while $\bar{\phi} > \bar{\phi}_{\rm f}$ **do**

  Evolve N, $\bar{\phi}$, $\bar{\pi}$.

  for *all modes k in the simulation* **do**

    if $k > \sigma a H$ **then**

      Evolve $\delta\phi_{\vec{k}}$, $\delta\phi'_{\vec{k}}$.

    else

      Evolve $\delta\phi_{\vec{k}}$, $\delta\phi'_{\vec{k}}$ to $k = \sigma a H$. Set the current kick coefficient from $\delta\phi_{\vec{k}}$, $\delta\phi'_{\vec{k}}$. Remove mode k from the simulation.

  if $k_{\rm next} \leq k_{\rm PBH}$ **then**

    if $k_{\rm next} \leq \alpha a H$ **then**

      Add mode $k = k_{\rm next}$ to the simulation. Set initial values for $\delta\phi_{\vec{k}}$, $\delta\phi'_{\vec{k}}$. Evolve $\delta\phi_{\vec{k}}$, $\delta\phi'_{\vec{k}}$ from $k = \alpha a H$. Set $k_{\rm next} = e^{1/32} k_{\rm next}$.

  else

    if $k_{\rm next} \leq \sigma a H$ **then**

      Set the current kick coefficient to zero.

  Add stochastic kick to $\bar{\phi}$, $\bar{\pi}$ using the current kick coefficient.