Stochastic inflation and primordial black holes

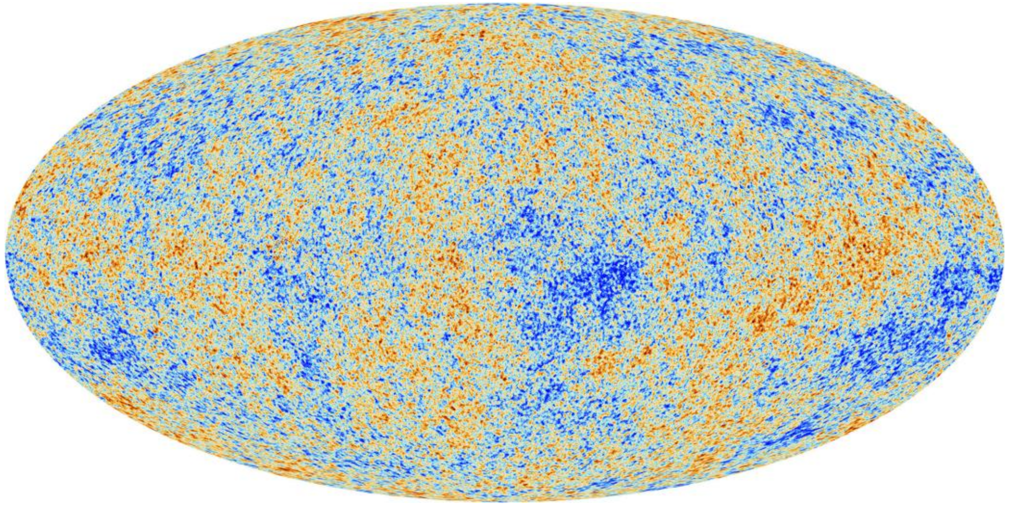
THEORETICAL COSMOLOGY MEETING,

5 JUNE 2026, UNIVERSITY OF GRONINGEN

EEMELI TOMBERG, UCLOUVAIN

Primordial perturbations

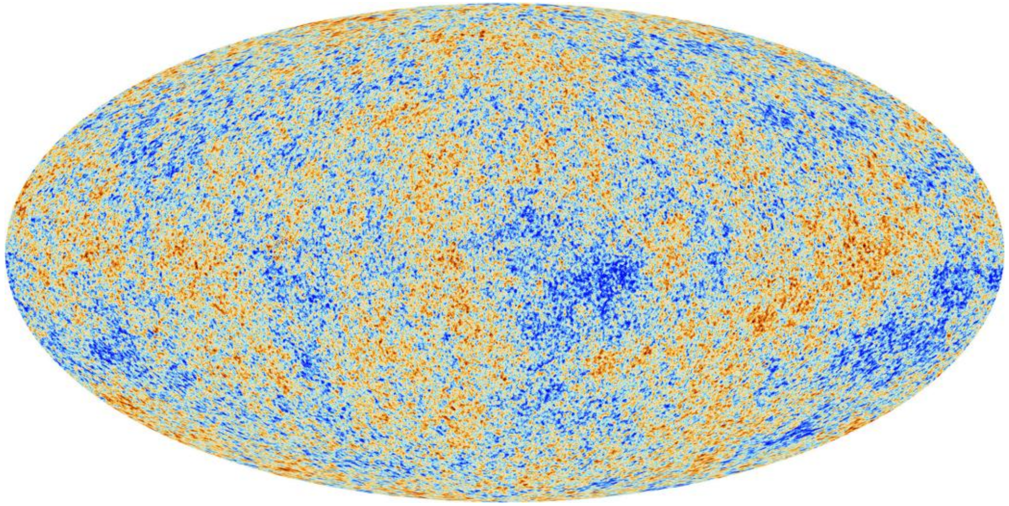
Long scales: weak perturbations
CMB, large-scale structures

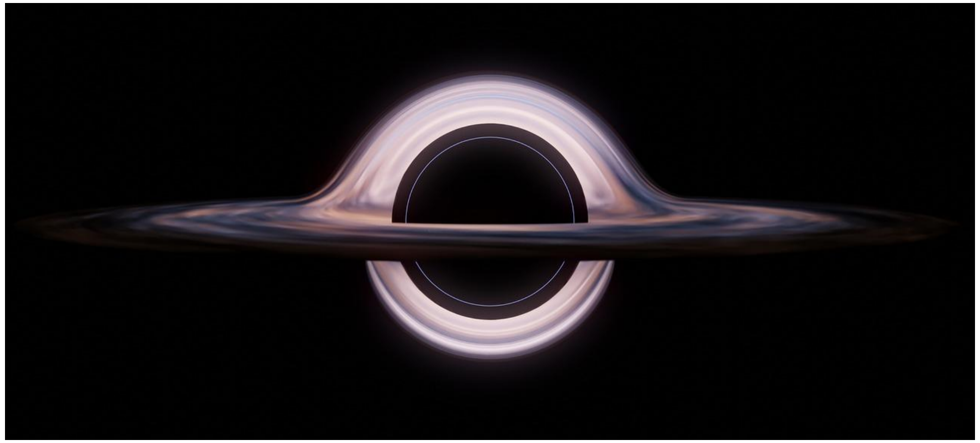


Primordial perturbations

Long scales: weak perturbations
CMB, large-scale structures

Short scales: ?
Primordial black holes?





Primordial black holes

Dark matter

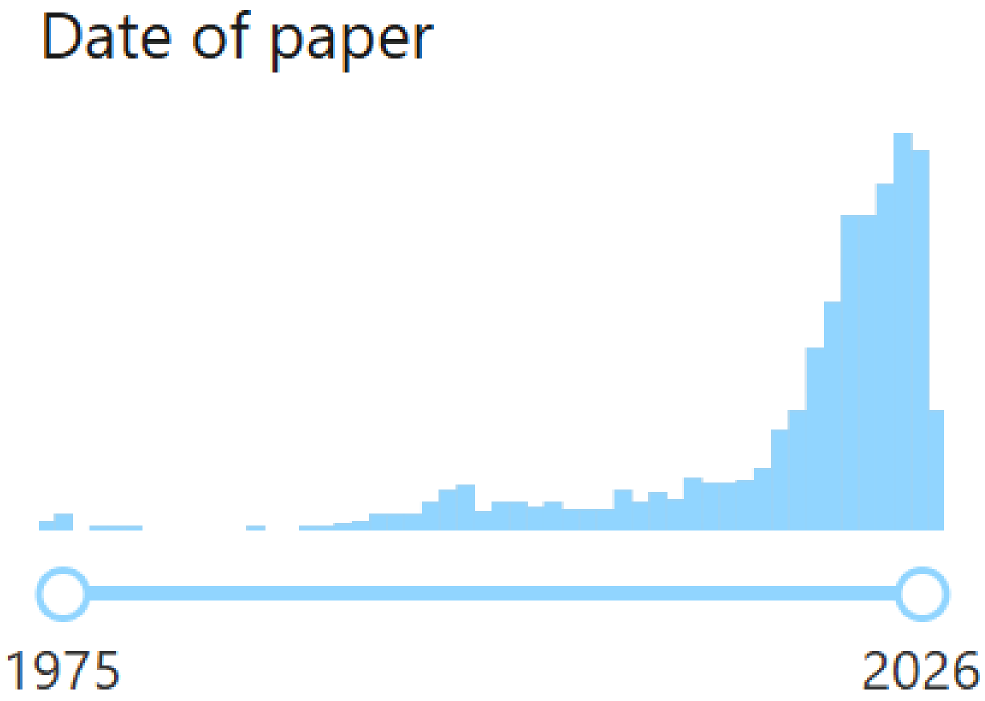
Seeds of supermassive BHs

GW source

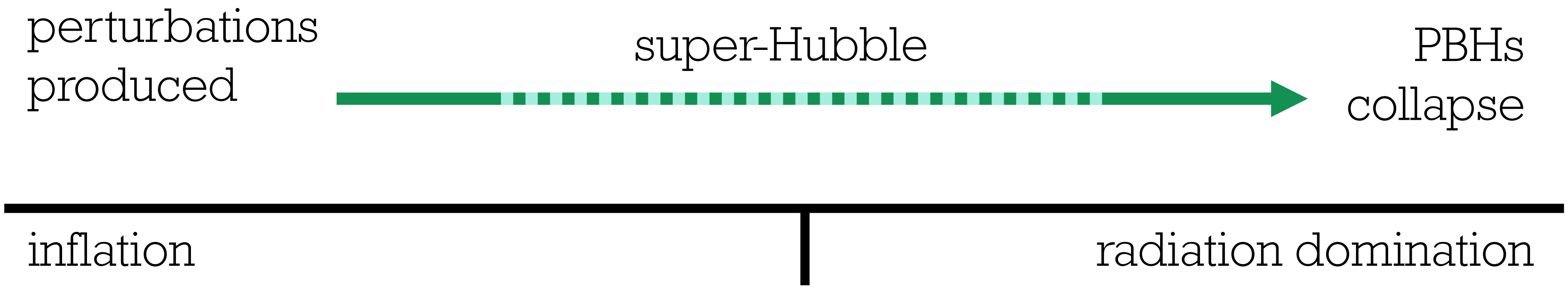
Primordial black holes

Zel'dovich and Novikov 1967

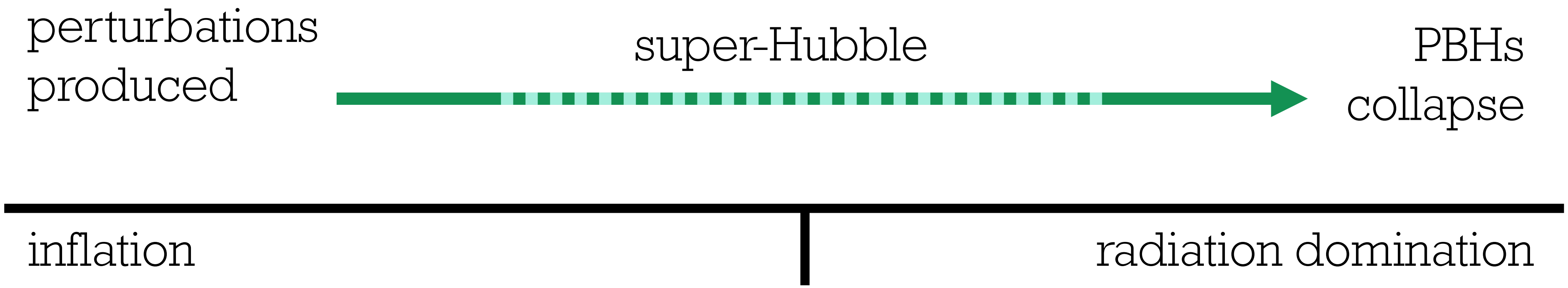
Hawking 1971 and Carr 1974



PBHs from inflation

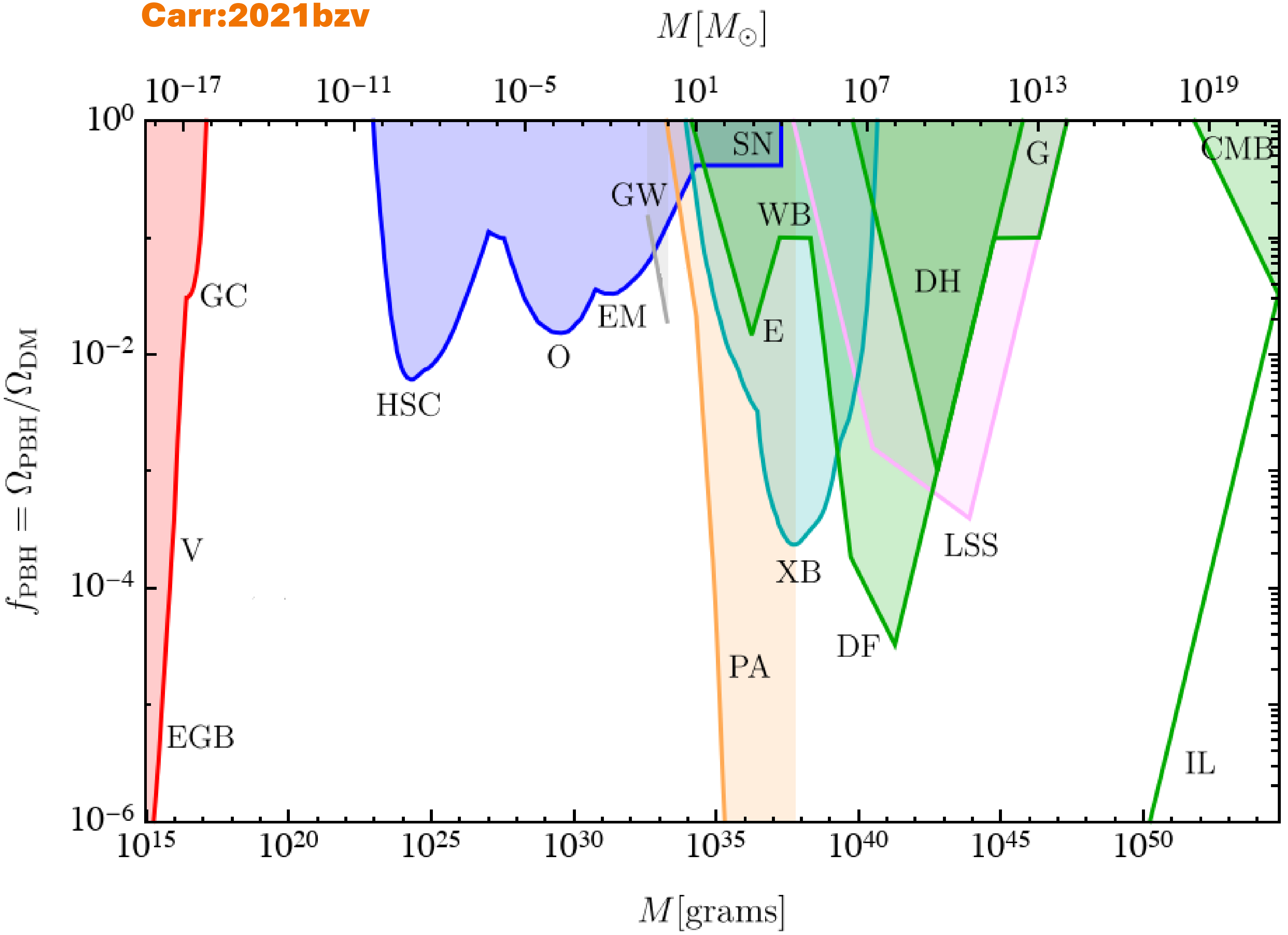


PBHs from inflation



Longer perturbations $\leftrightarrow$ larger PBH mass

Carr:2021bzv



Inflationary perturbations

Background

Equations of motion:

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0\,, \qquad 3H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

Slow-roll parameters:

$$\epsilon_1 \equiv -\partial_N \ln H\,, \qquad \epsilon_2 \equiv \partial_N \ln \epsilon_1$$

Inflation?

Slow-roll?

Perturbations

Field perturbations:

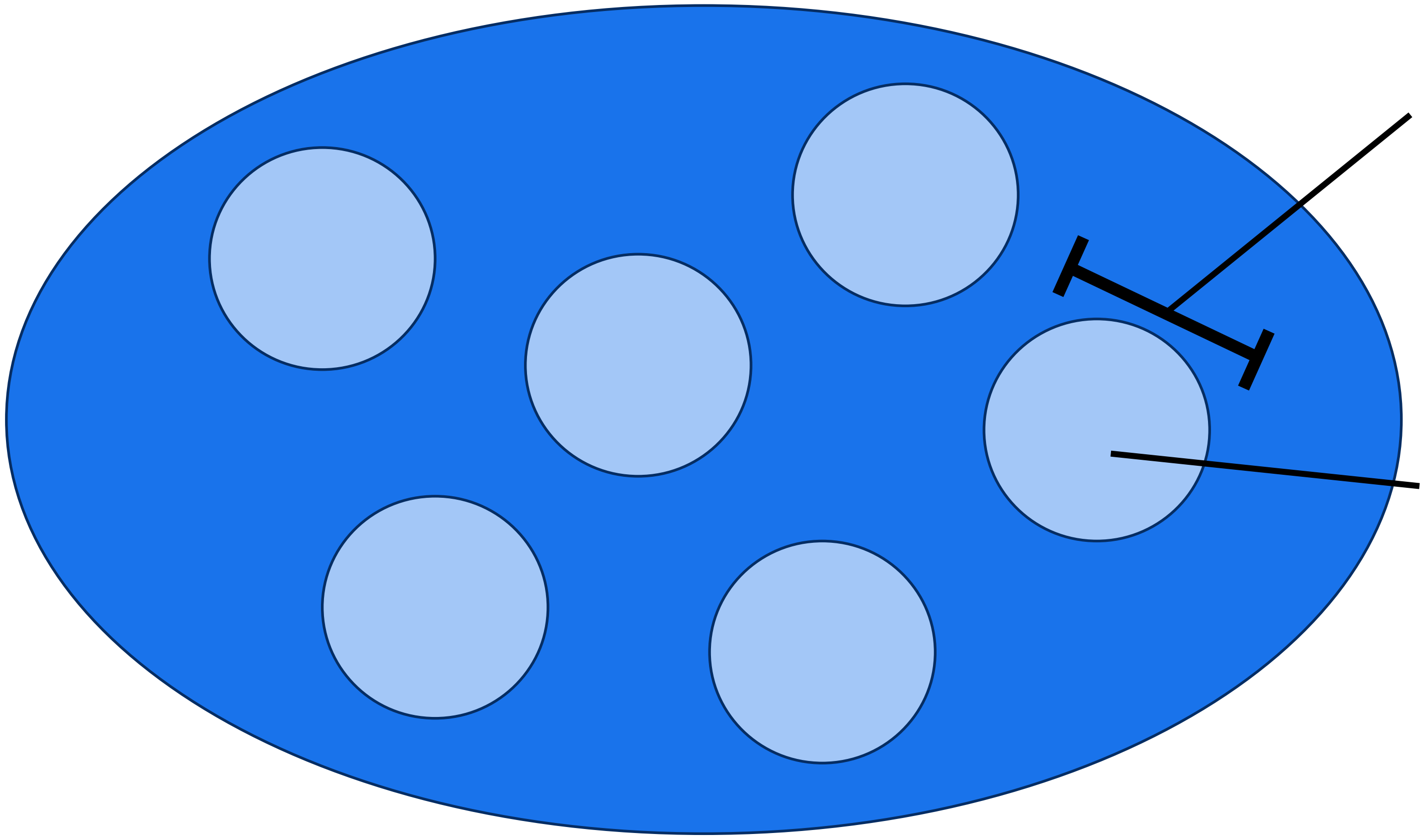
$$\delta\ddot{\phi} + 3H\delta\dot{\phi} + \left[\frac{k^2}{a^2} + V''(\phi) - \frac{1}{a^3}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{a^3}{H}\dot{\phi}^2\right)\right]\delta\phi = 0$$

Curvature perturbation:

$$\zeta_k = \frac{\delta\phi_k}{\sqrt{2\epsilon_1}} \quad \longleftarrow \sim H \qquad\qquad \mathcal{P}_\zeta(k) = \frac{k^3}{2\pi^2}|\zeta_k|^2$$

Beyond linear perturbations

Separate universe approximation:



ΔN approximation: $\Delta N = N - \langle N \rangle = \zeta$

$$\ddot{\phi} - \frac{1}{a^2}\nabla^2\phi + 3H\dot{\phi} + V'(\phi) = 0$$

Linear perturbations:
small-$\delta\phi$ expansion

$$\ddot{\phi} - \frac{1}{a^2}\nabla^2\phi + 3H\dot{\phi} + V'(\phi) = 0$$

$$\delta\ddot{\phi} \qquad \delta\phi \qquad \delta\dot{\phi} \qquad V''(\phi)\delta\phi$$

+ metric perts

Separate universe:
small-$\frac{\nabla^2}{a^2H^2}$ expansion

$$\ddot{\phi} - \frac{1}{a^2}\nabla^2\phi + 3H\dot{\phi} + V'(\phi) = 0$$

Divide the field in two

$$\phi_{\rm tot} = \phi + \delta\phi$$

$$\int_{k<k_\sigma} \frac{\mathrm{d}k^3}{(2\pi)^{2/3}} \phi_k(t) e^{-i\vec{k}\cdot\vec{x}}$$

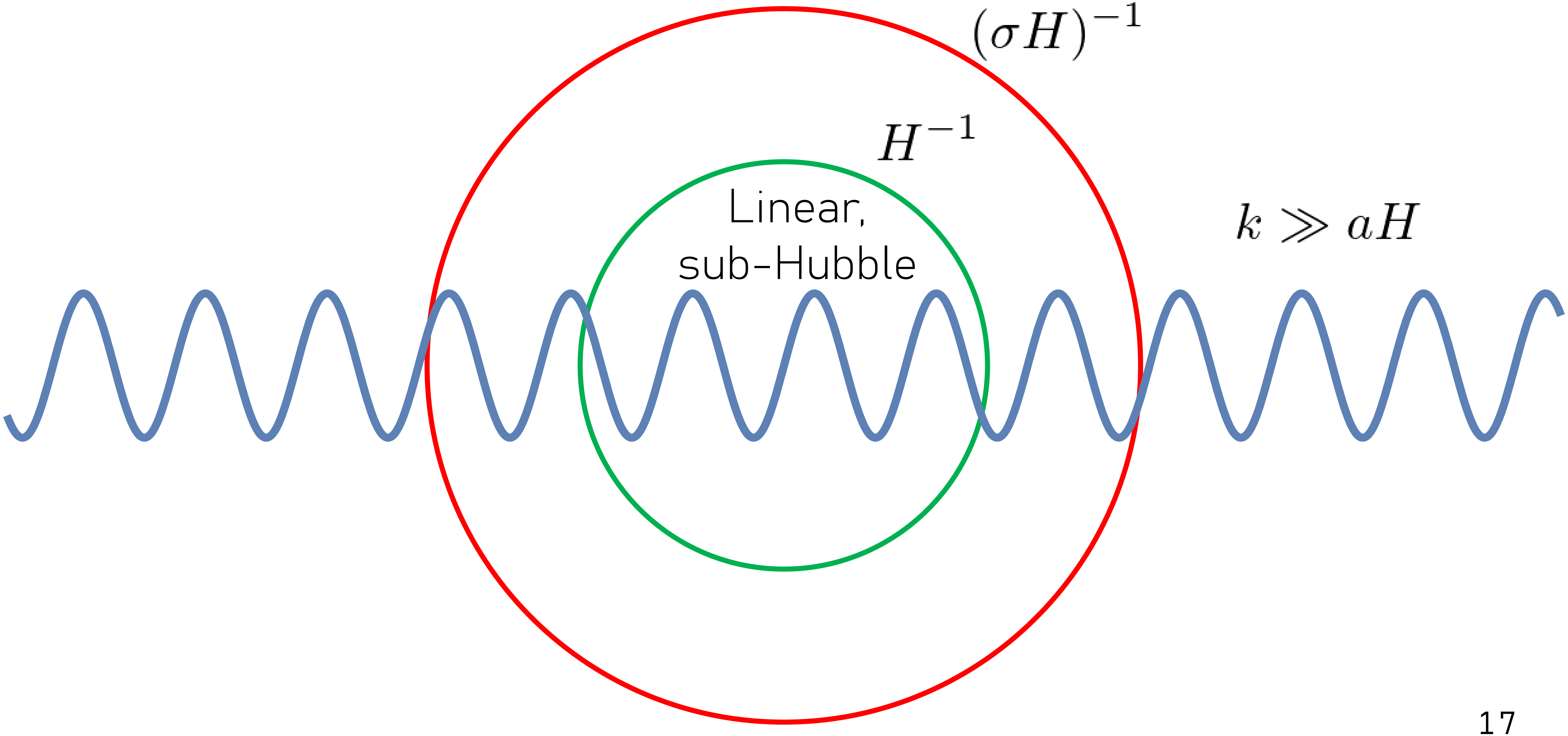
Separate universe

$$\int_{k>k_\sigma} \frac{\mathrm{d}k^3}{(2\pi)^{2/3}} \phi_k(t) e^{-i\vec{k}\cdot\vec{x}}$$

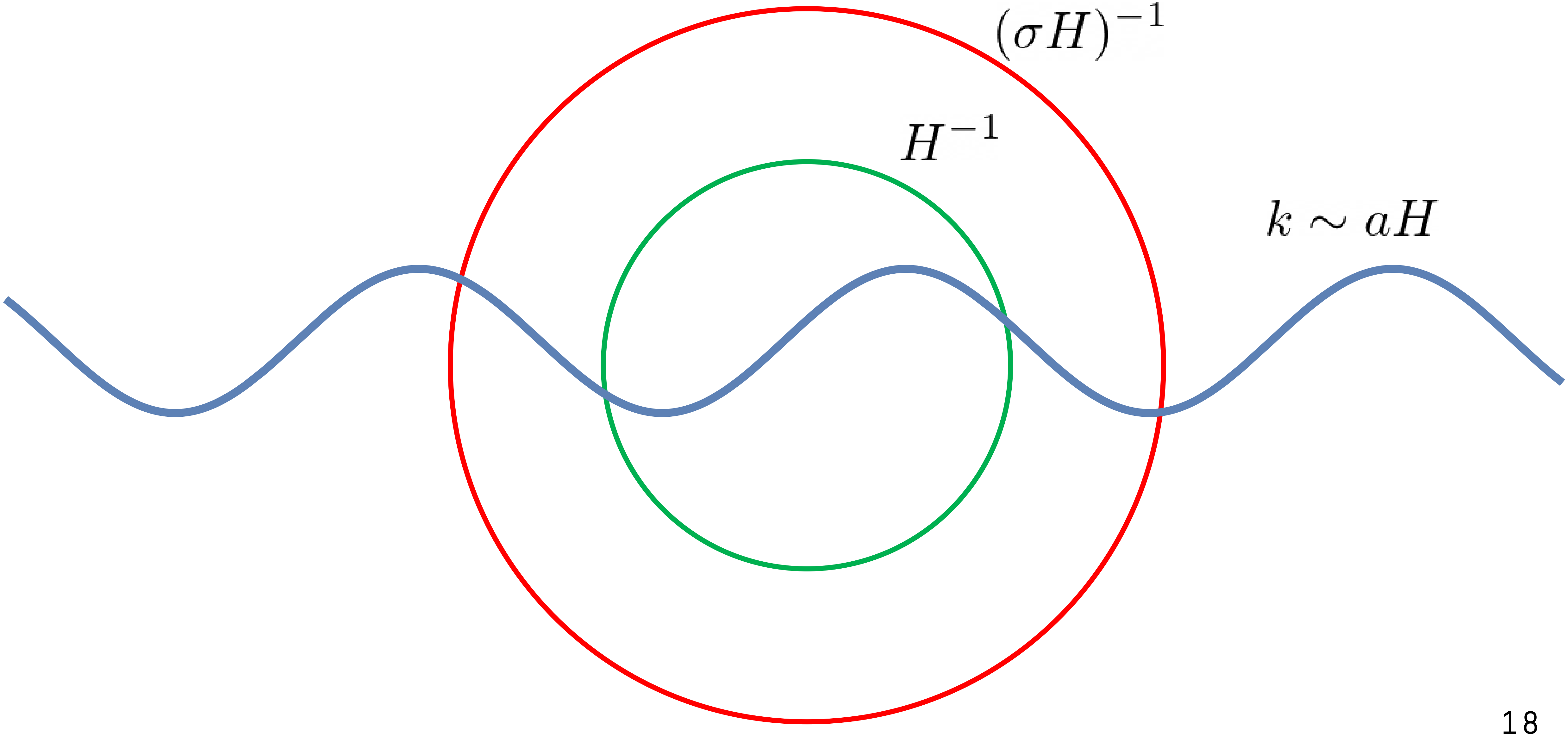
Linear perturbations

Coarse-graining scale: $k = k_\sigma \equiv \sigma a H$

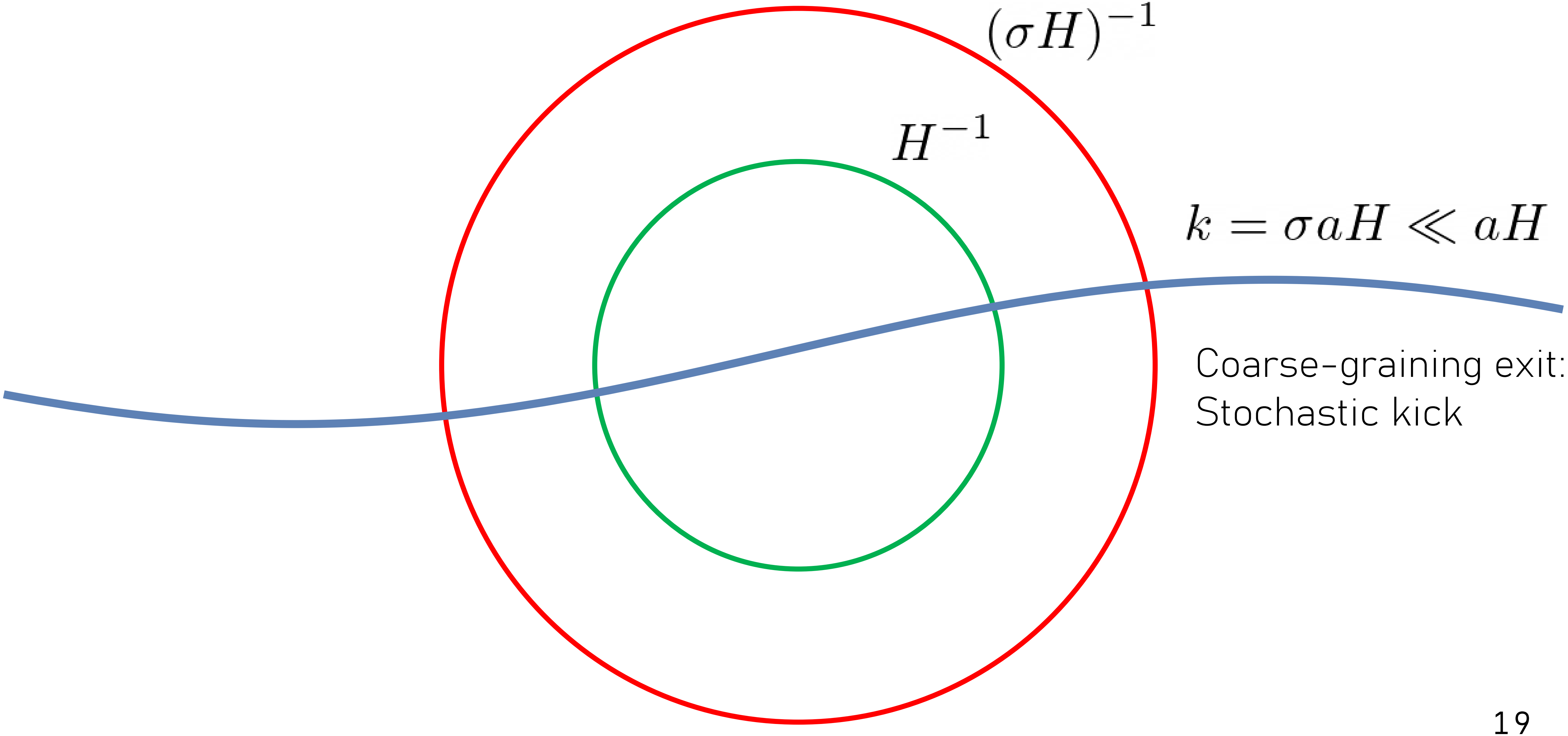
Drifting modes: stochasticity



Drifting modes: stochasticity



Drifting modes: stochasticity



Stochastic inflation

$$\partial_N \phi = \pi + \xi_\phi \,, \quad \partial_N \pi = -\left(3 - \frac{1}{2}\pi^2\right)\left(\pi + \frac{V'(\phi)}{V(\phi)}\right) + \xi_\pi$$

e-folds as time variable

$$\langle \xi_X(N) \xi_Y(\tilde{N}) \rangle = \mathcal{P}_{XY}(N, k_\sigma)\delta(N - \tilde{N})$$

↑

de Sitter result / numerical computation

Stochastic inflation

Slow roll:

$$\partial_N \phi = -\frac{V'(\phi)}{V(\phi)} + \xi_\phi$$

$$3H^2(\phi) = V(\phi)$$

$$\left\langle \xi_\phi(N)\xi_\phi(\tilde{N}) \right\rangle = \frac{H^2(\phi)}{4\pi^2}\delta(N-\tilde{N})$$

Figueroa:2020jkf

Initial ϕ

stochastic evolution

Switch noise off

classical evolution

Final ϕ

$N = 0$

Final coarse-graining scale

$k = \sigma a H$

final $N = N_{<k}$

Radial profiles

Raatikainen:2023bzk

$$\Delta N_{<k} = N_{<k} - \langle N_{<k} \rangle$$
$$= \zeta_{<k}(\mathbf{x}) \equiv \int \frac{\mathrm{d}^3 p}{(2\pi)^{3/2}} \zeta_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{x}} \theta(k-p)$$

Radial profiles

Raatikainen:2023bzk

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$$= \zeta_{<k}(\mathbf{x}) \equiv \int \frac{\mathrm{d}^3 p}{(2\pi)^{3/2}} \zeta_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{x}} \theta(k-p)$$

Vary coarse-graining scale for same patch

Radial profiles

Raatikainen:2023bzk

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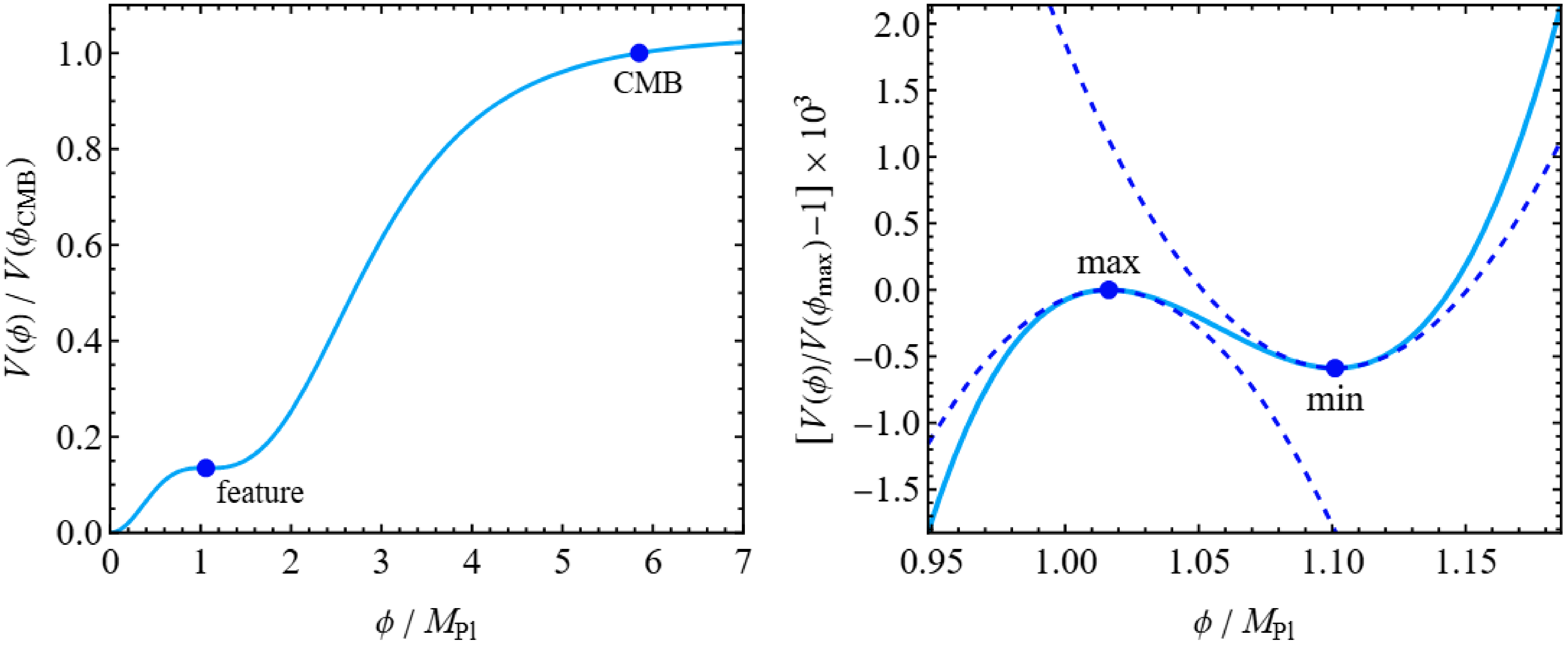
$$= \zeta_{<k}(\mathbf{x}) \equiv \int \frac{\mathrm{d}^3 p}{(2\pi)^{3/2}} \zeta_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{x}} \theta(k-p)$$

Vary coarse-graining scale for same patch

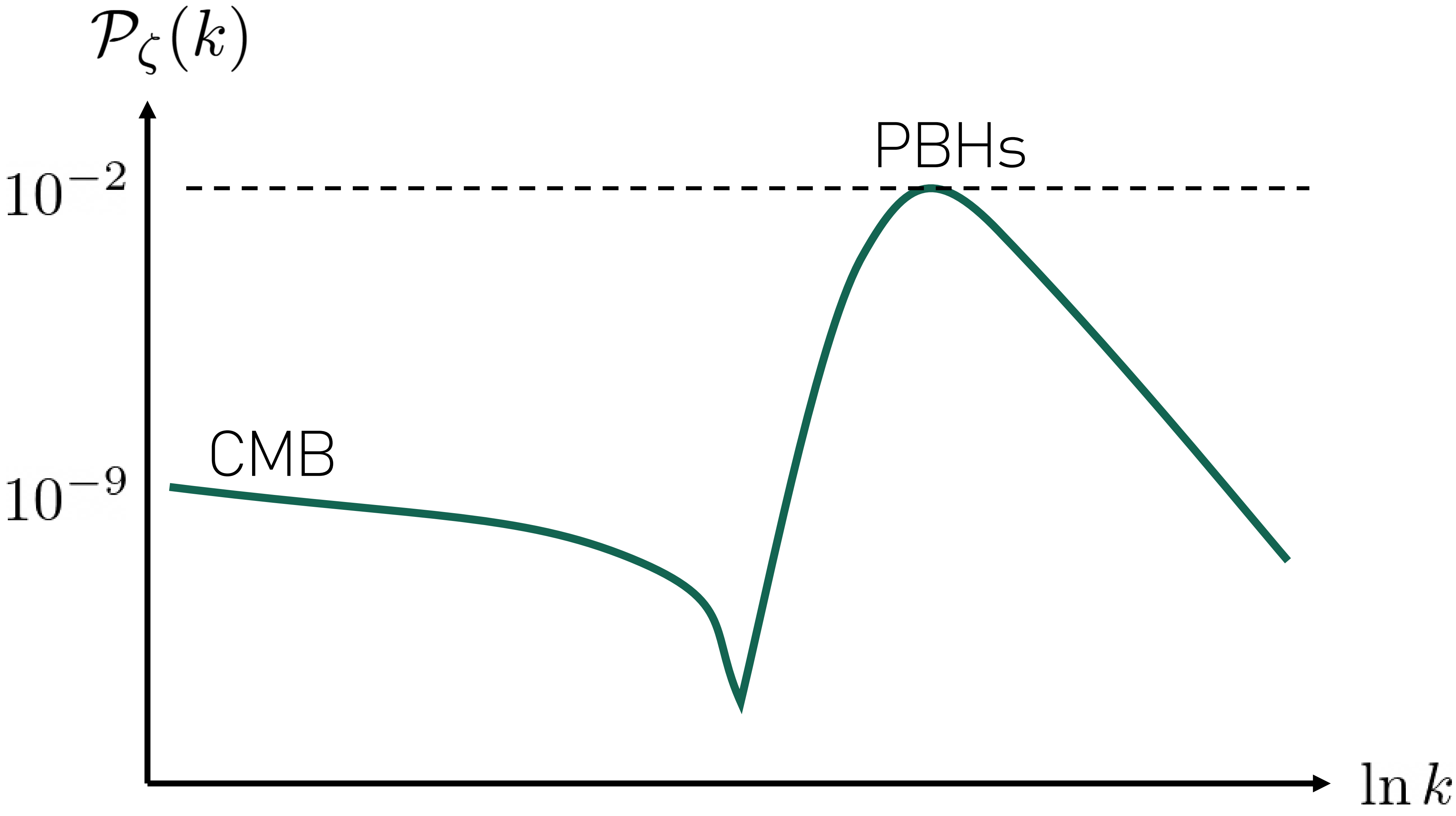
Obtain: $\zeta_k = \sqrt{\frac{\pi}{2}} \frac{1}{k^3} \frac{\mathrm{d}\zeta_{<k}}{\mathrm{d}\ln k}$ ➡ $\zeta(r) = \int_0^\infty \frac{\mathrm{d}k}{k} \frac{\mathrm{d}\zeta_{<k}}{\mathrm{d}\ln k} \frac{\sin(kr)}{kr}$

Primordial black hole formation

Inflection point models

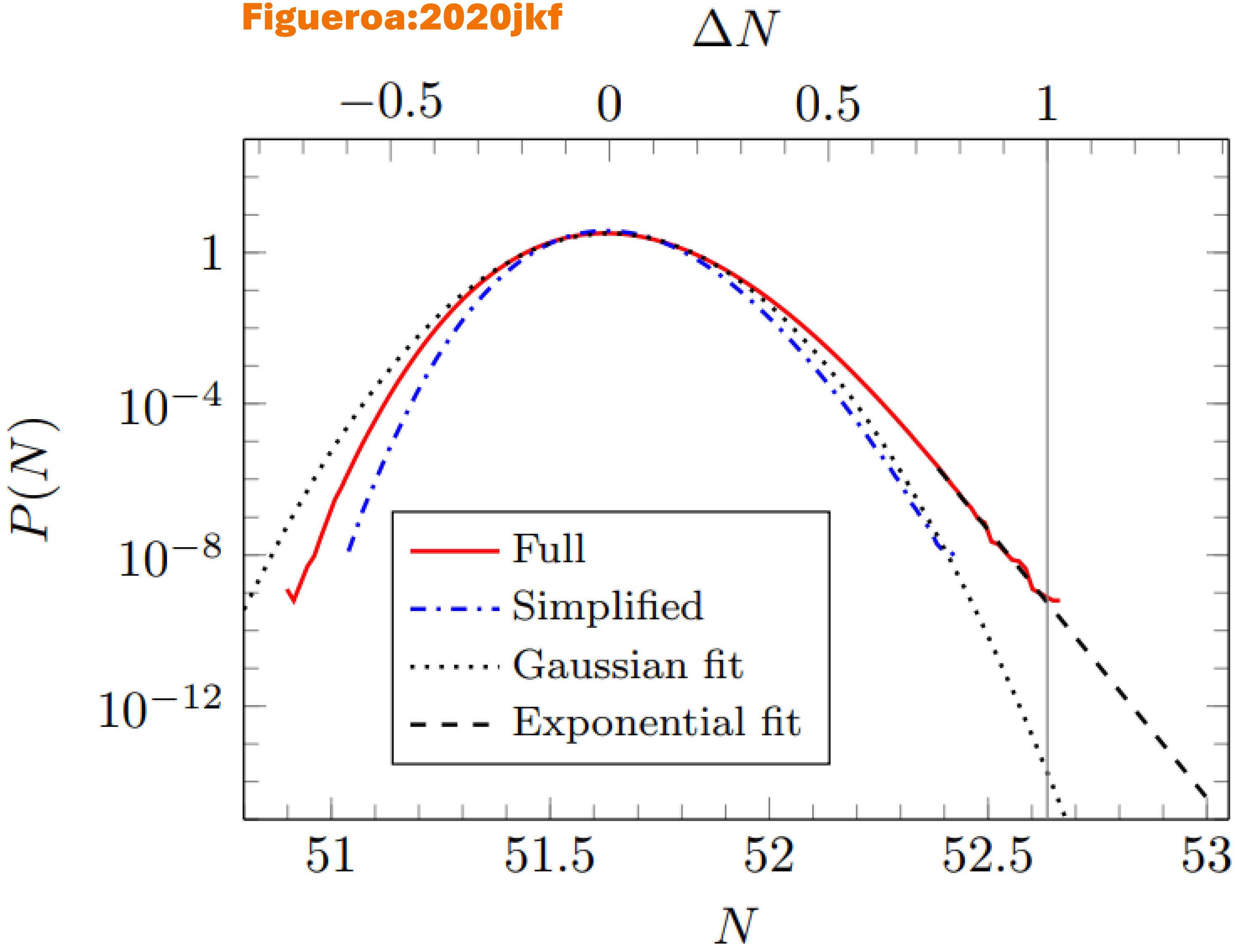


Inflection point models

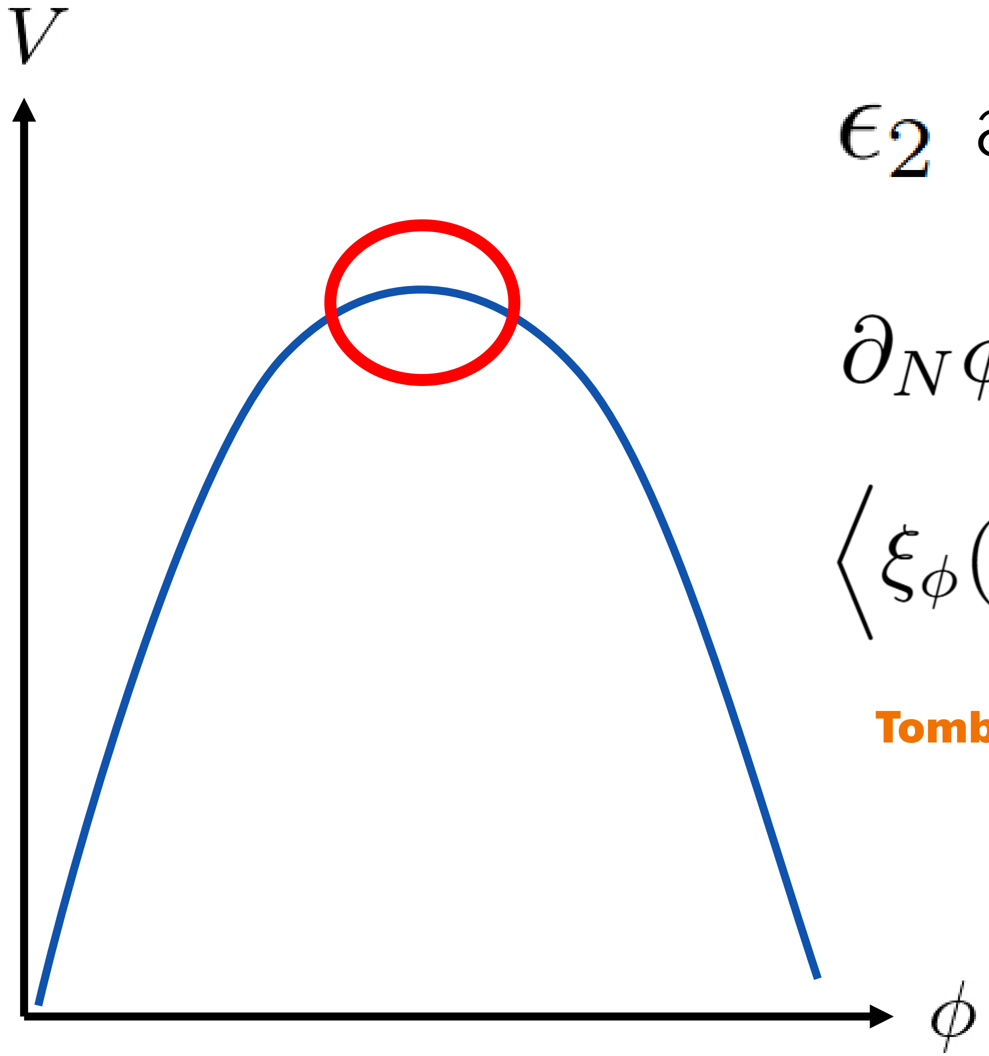


Exponential tails

Figueroa:2020jkf



Simplification: constant roll



ϵ_2 approx. constant

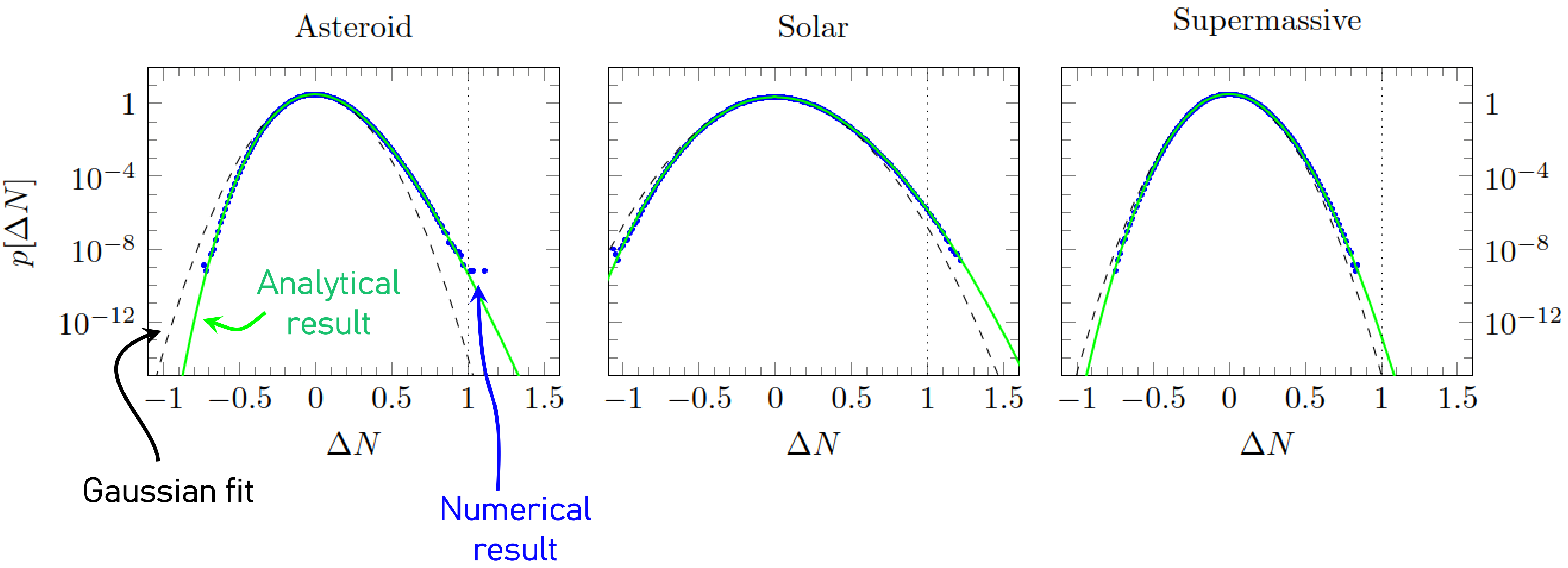
$$\partial_N \phi = \frac{\epsilon_2}{2}\phi + \xi_\phi$$

$$\left\langle \xi_\phi(N)\xi_\phi(\tilde{N}) \right\rangle = \mathcal{P}_\phi(N, k_\sigma)\delta(N - \tilde{N})$$

Tomberg:2023kli

Simplification: constant roll

Tomberg:2023kli



Which perturbations collapse?

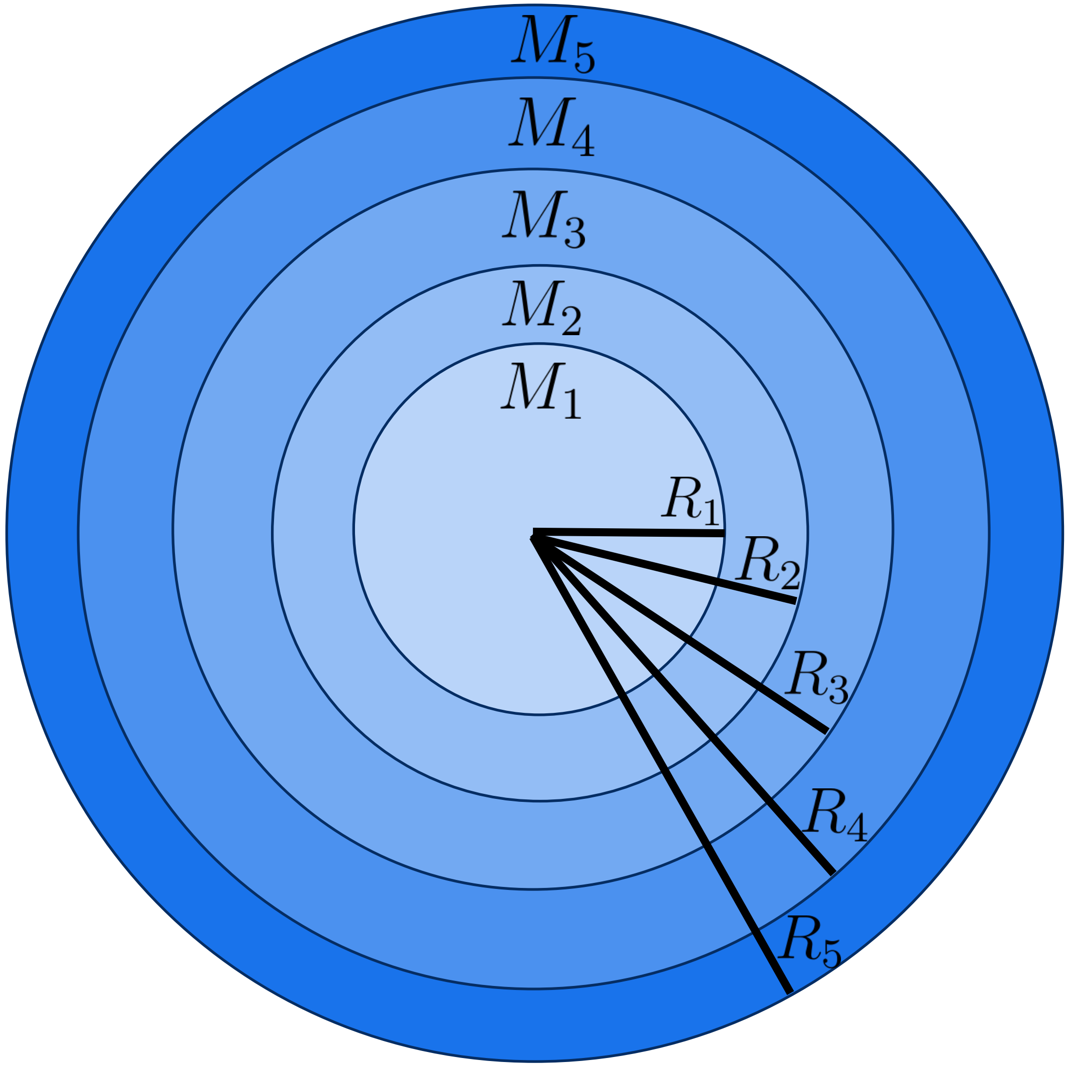
Numerical relativity simulations

Compaction function:

$$\mathcal{C}(r,t) = 2\frac{\delta M(r,t)}{R(r,t)}$$

Collapse:

$$\mathcal{C}_{\rm max} > \mathcal{C}_{\rm th} = \delta_c$$



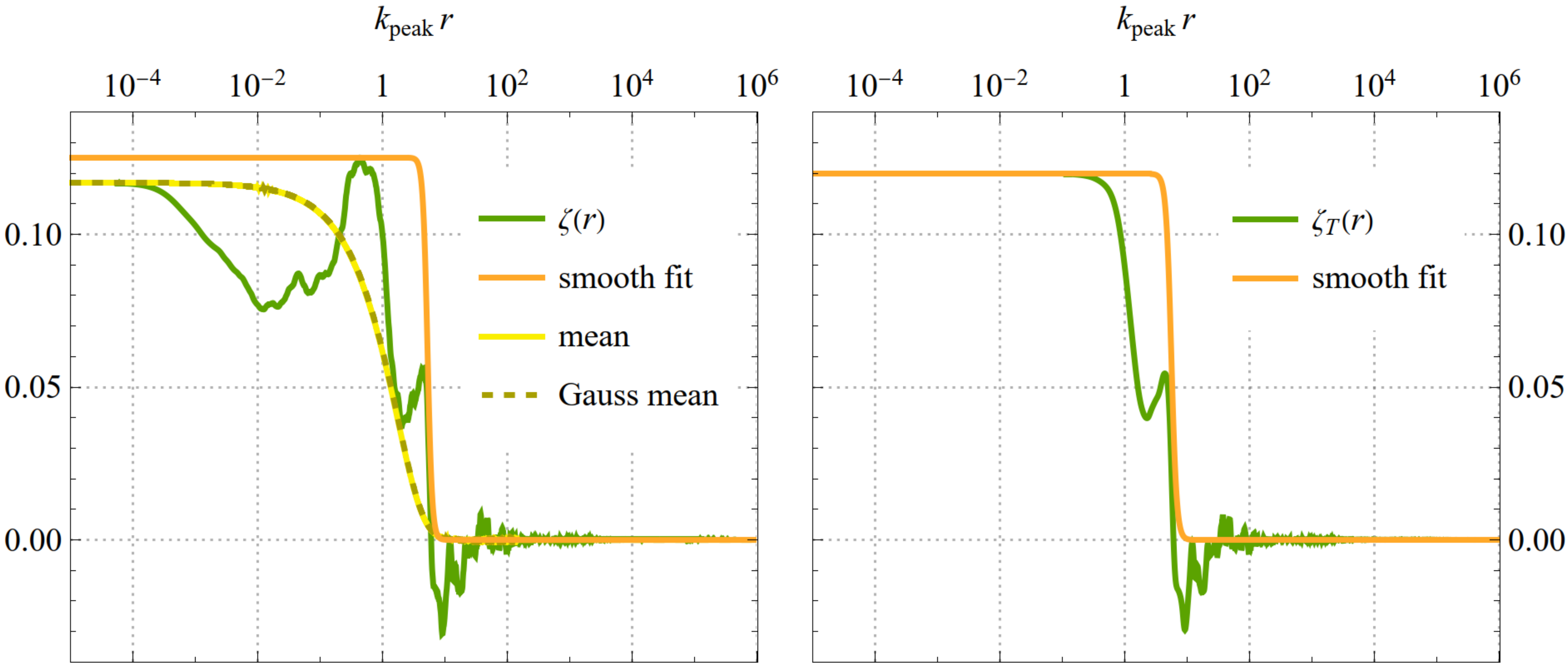
Compaction versus curvature

Super-Hubble:

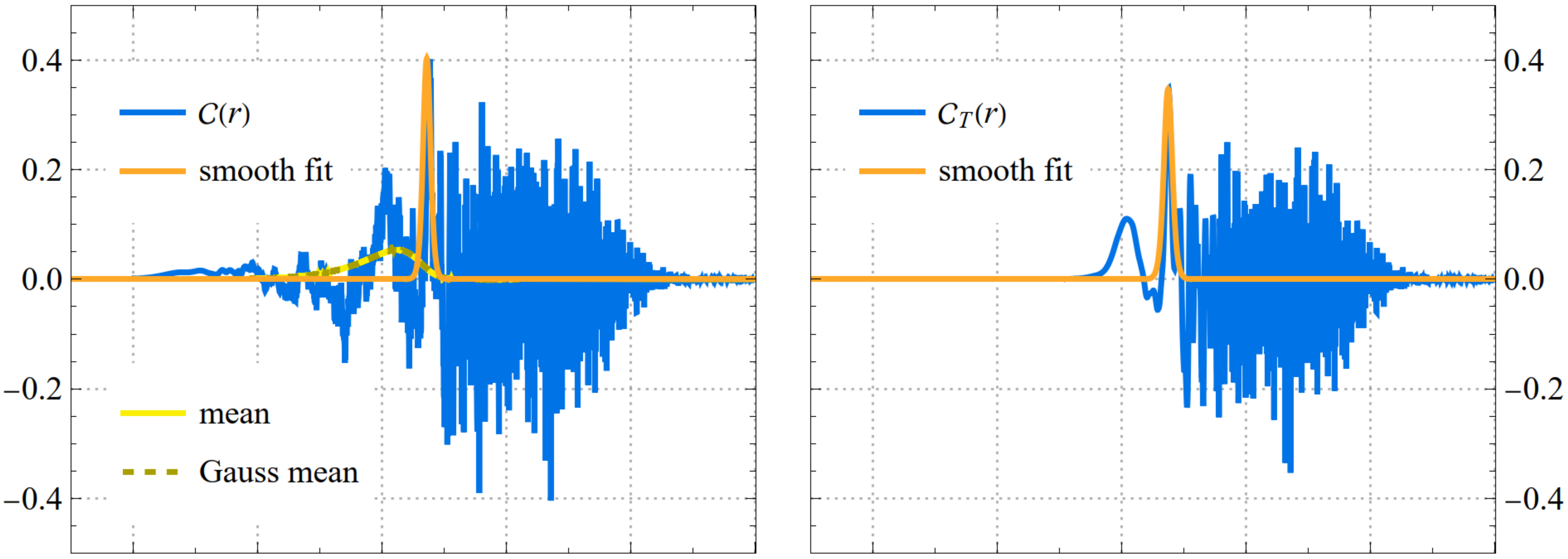
$$\mathcal{C}(r) = \frac{2}{3}\left(1 - [1 + r\zeta'(r)]^2\right)$$

Note assumption of spherical symmetry

Raatikainen:2025gpd



Raatikainen:2025gpd



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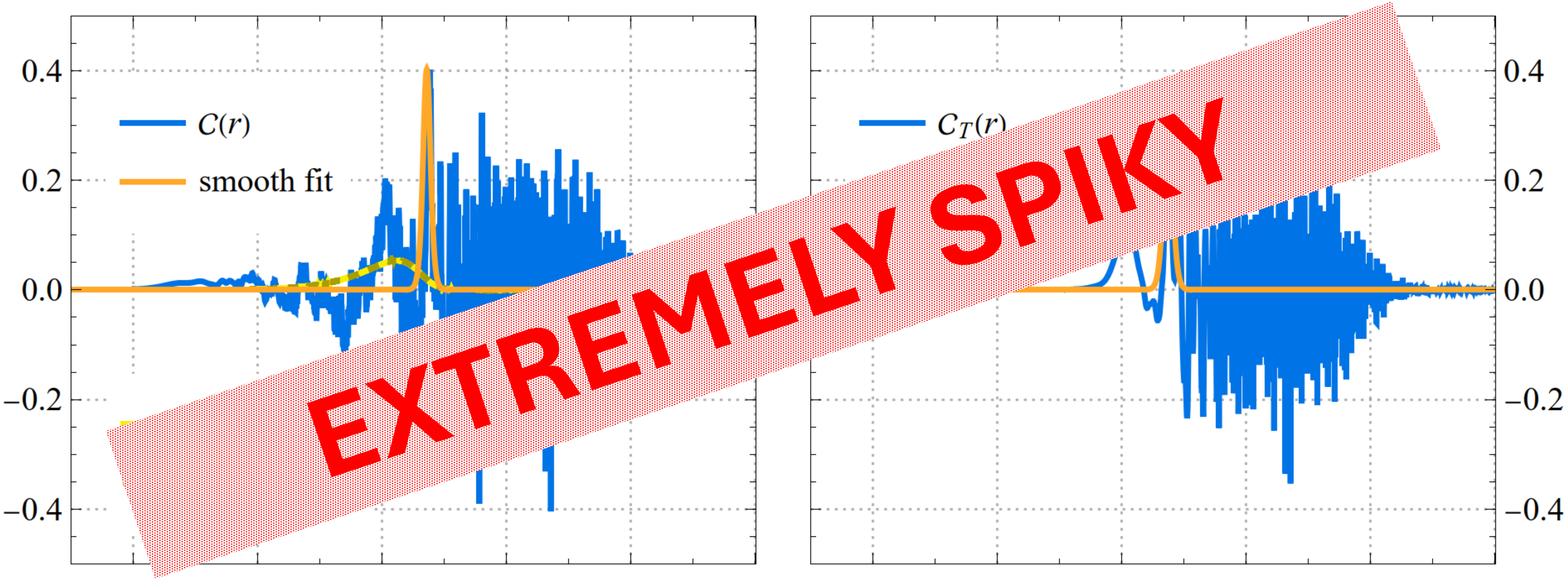

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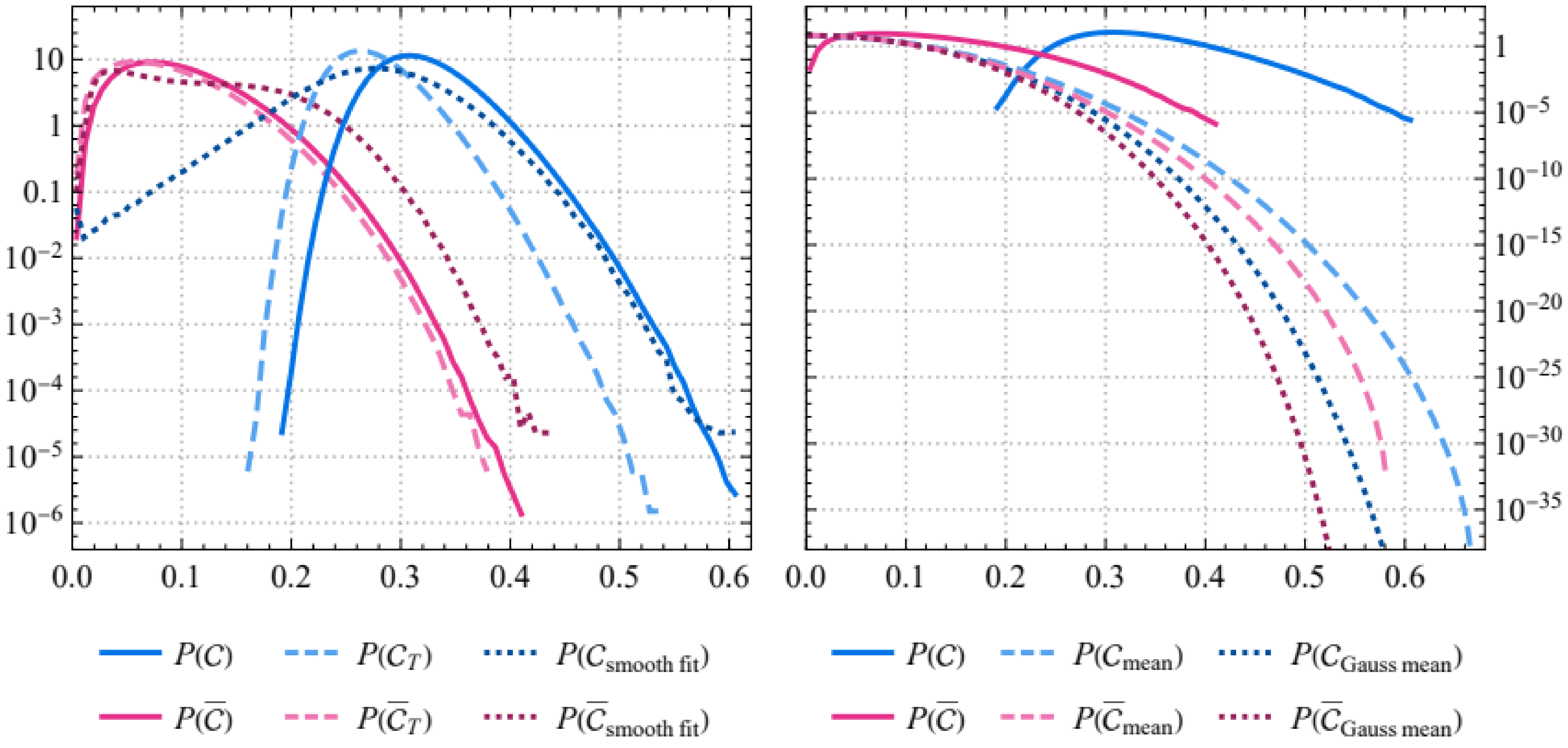

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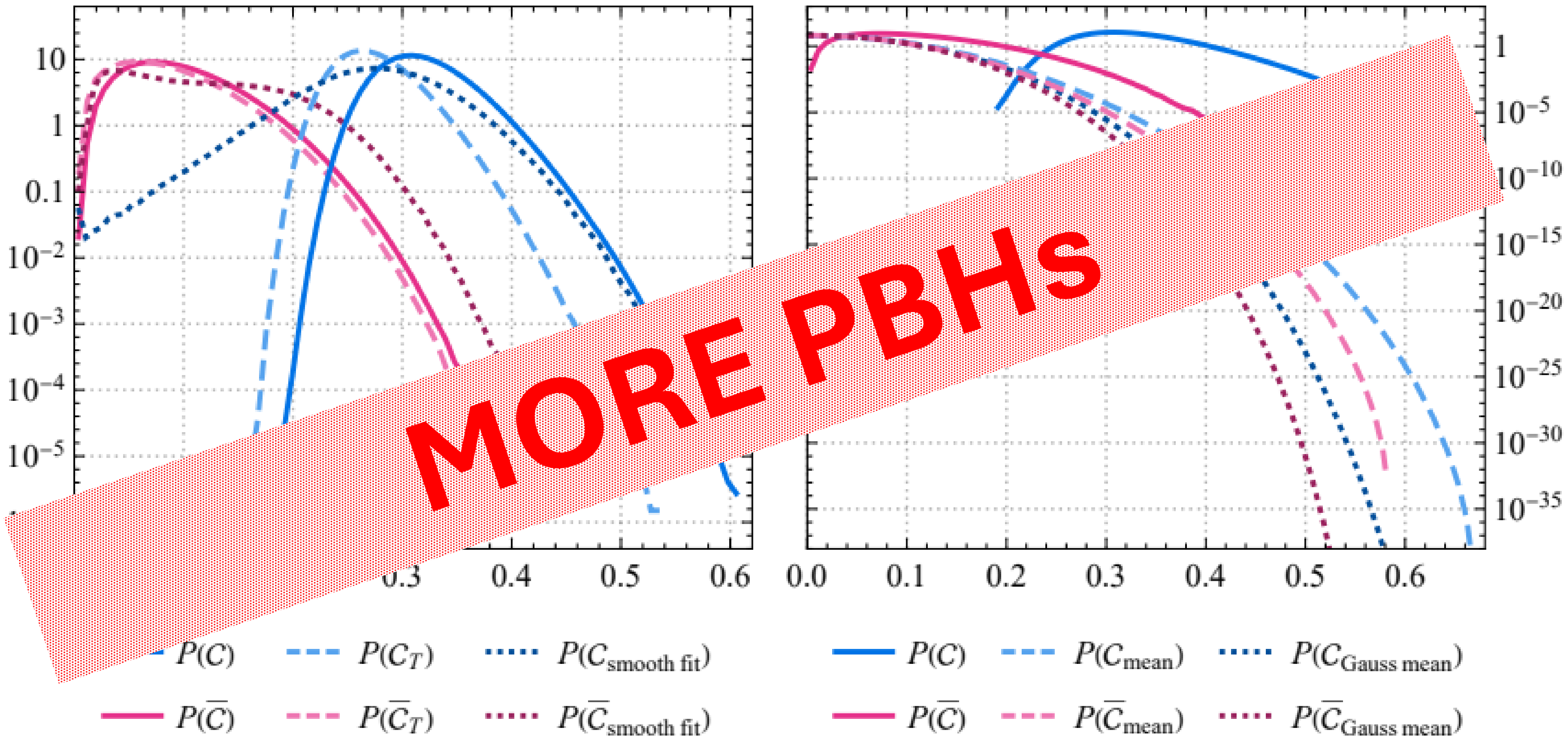
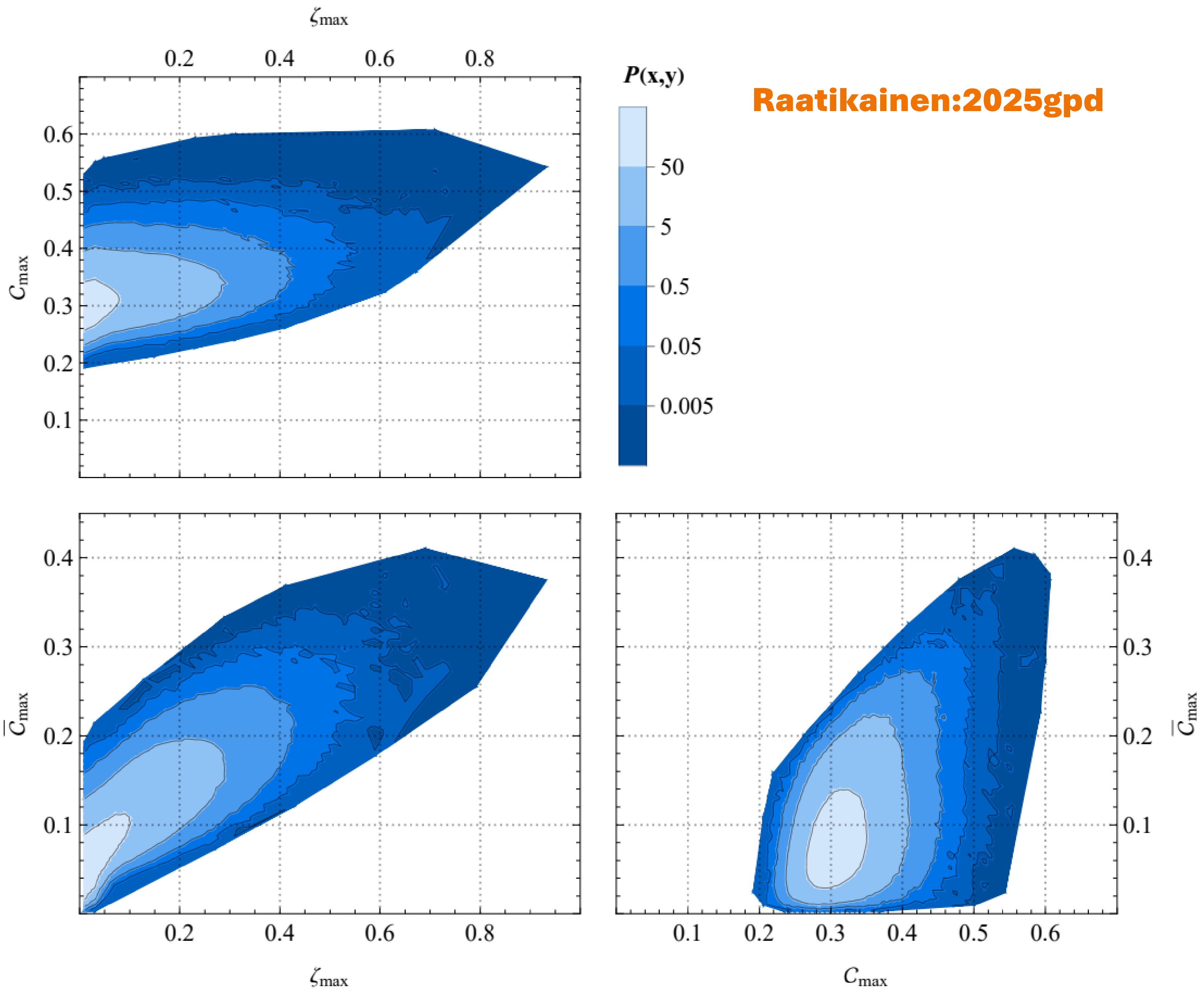
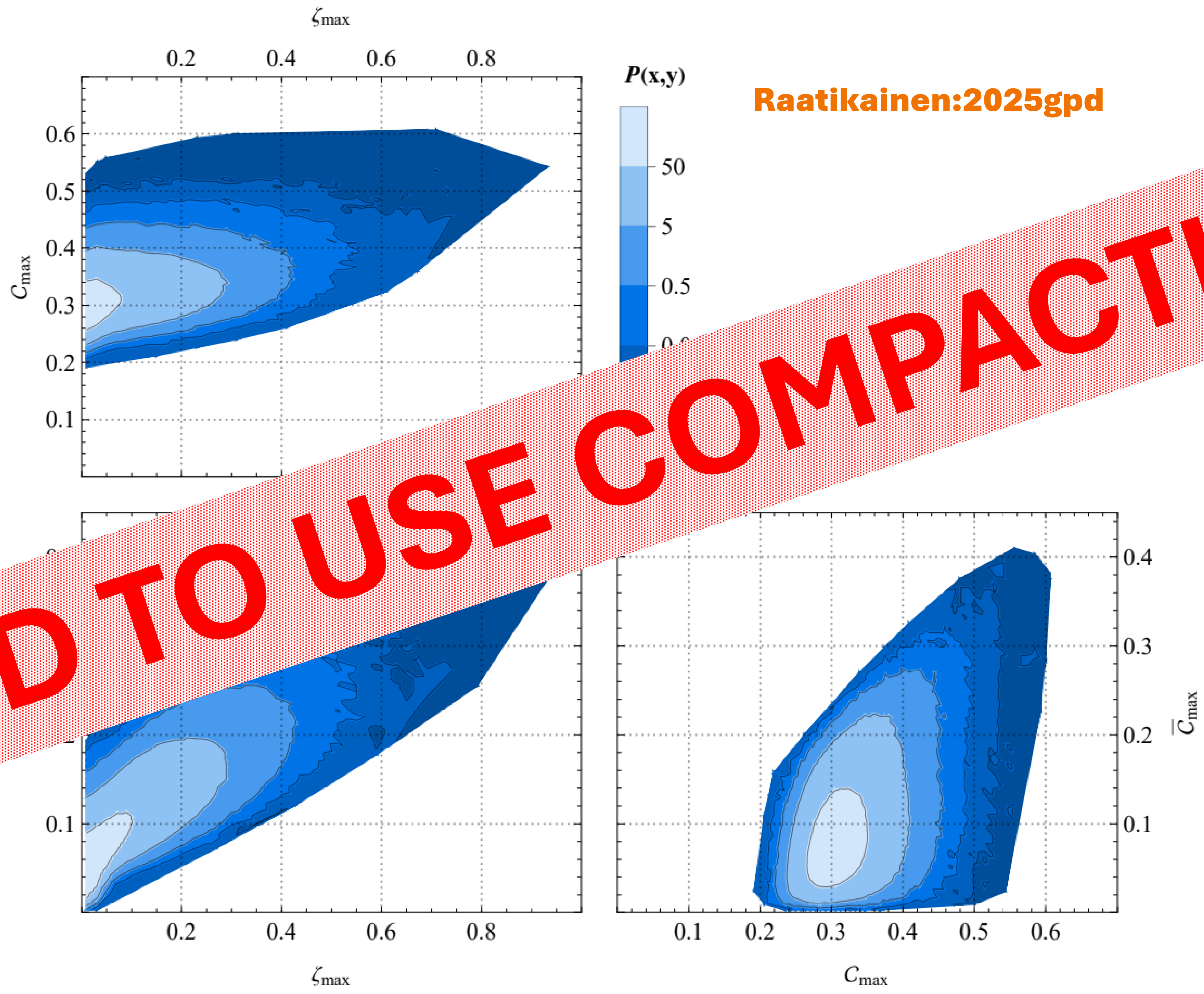


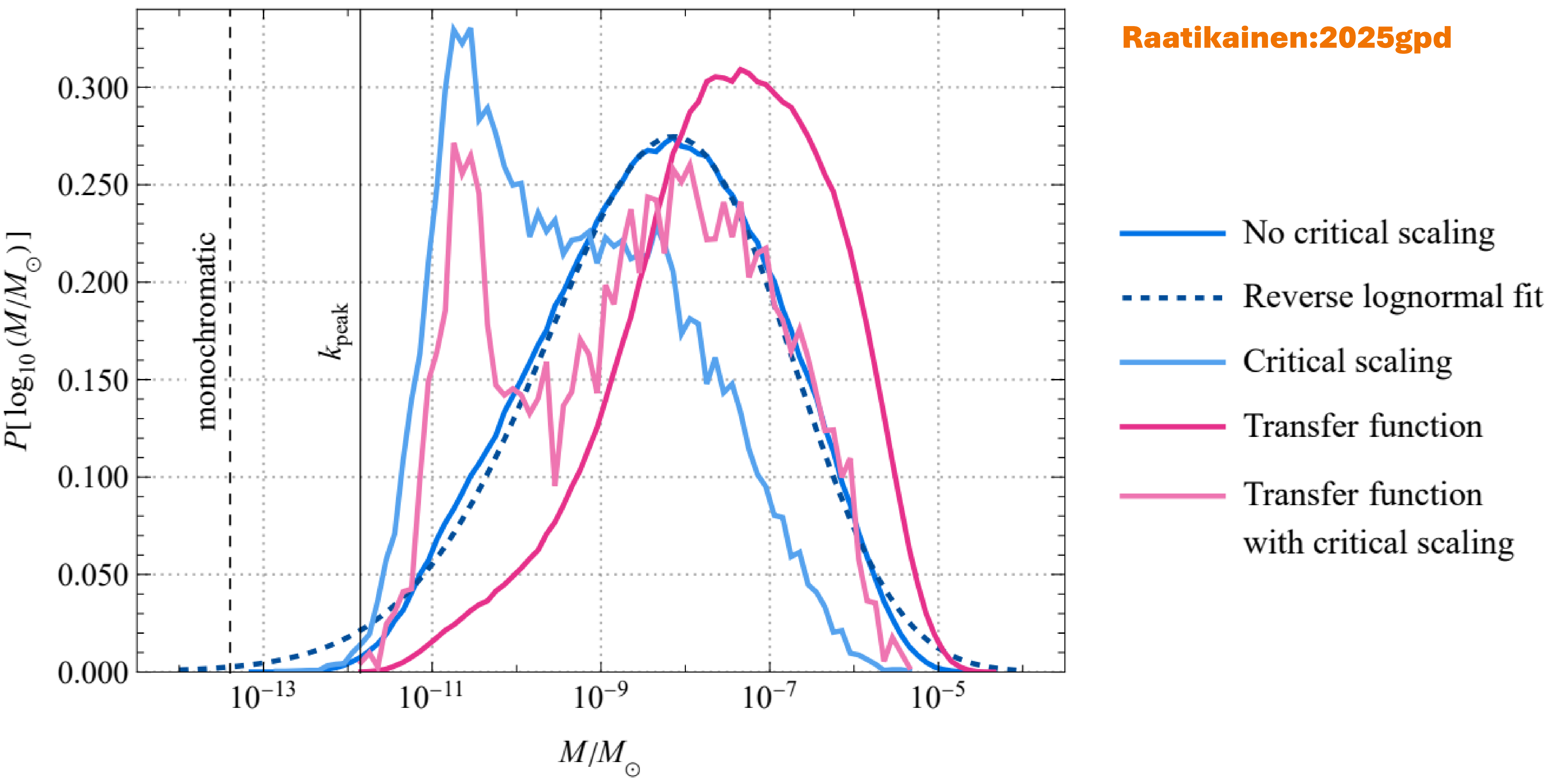


ζ_{max}
0.2
0.4
0.6
0.8
C_{max}
0.6
0.5
0.4
0.3
0.2
0.1
$P(x,y)$
50
5
0.5
0.05
0.005
Raatikainen:2025gpd
$\overline{C}_{max}$
0.4
0.3
0.2
0.1
0.6



Raatikainen:2025gpd

NEED TO USE COMPACTION



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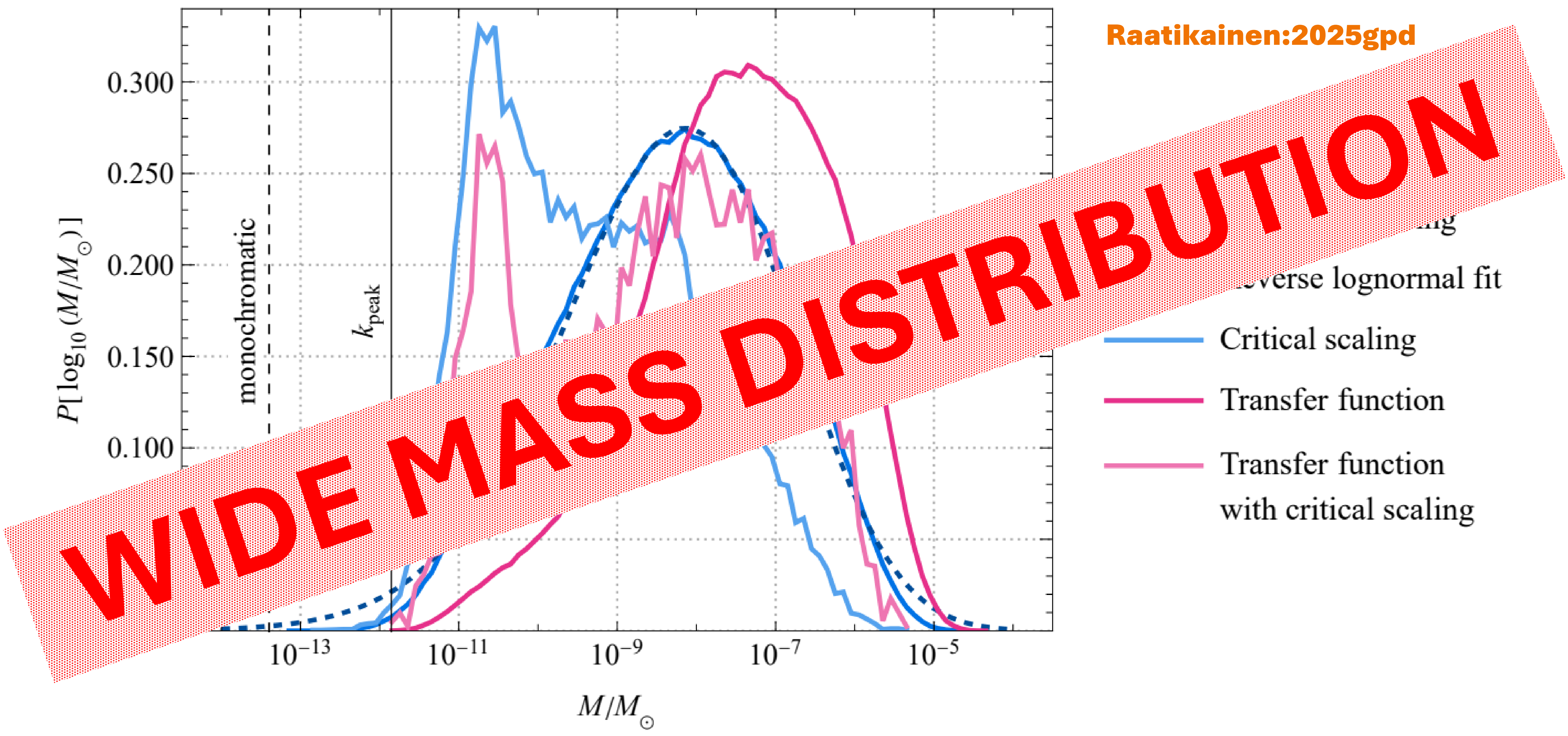


WIDE MASS DISTRIBUTION
Raatikainen:2025gpd
0.300
0.250
0.200
0.150
0.100
monochromatic
$k_{\rm peak}$
$P[\log_{10}(M/M_\odot)]$
10^{-13}
10^{-11}
10^{-9}
10^{-7}
10^{-5}
$M/M_\odot$
Critical scaling
Transfer function
Transfer function with critical scaling

Open questions

Behaviour at large radius?

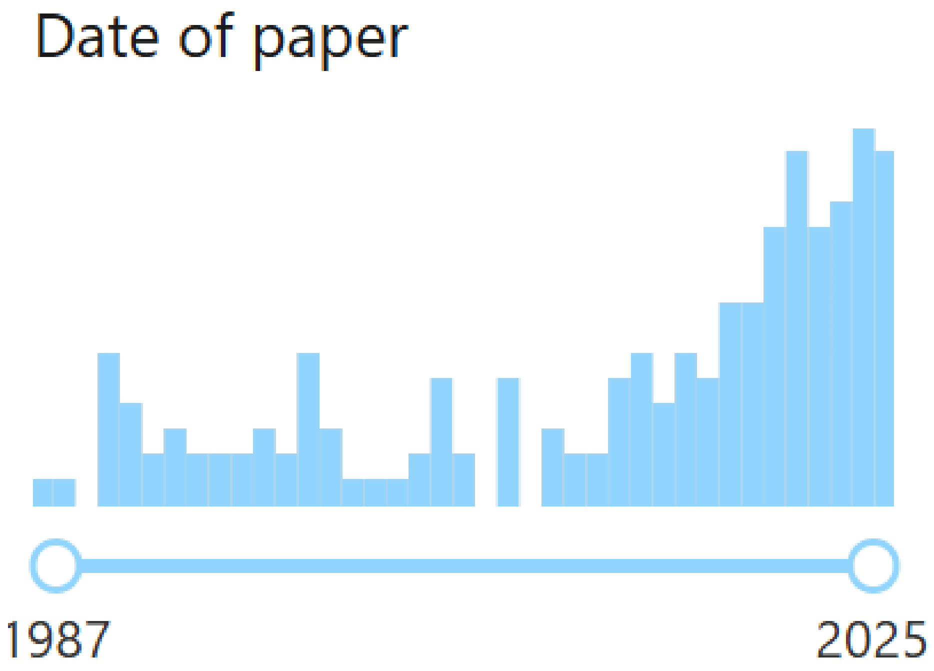
Correct collapse criterion for spiky profiles?

Redo collapse simulations?

Stochastic inflation

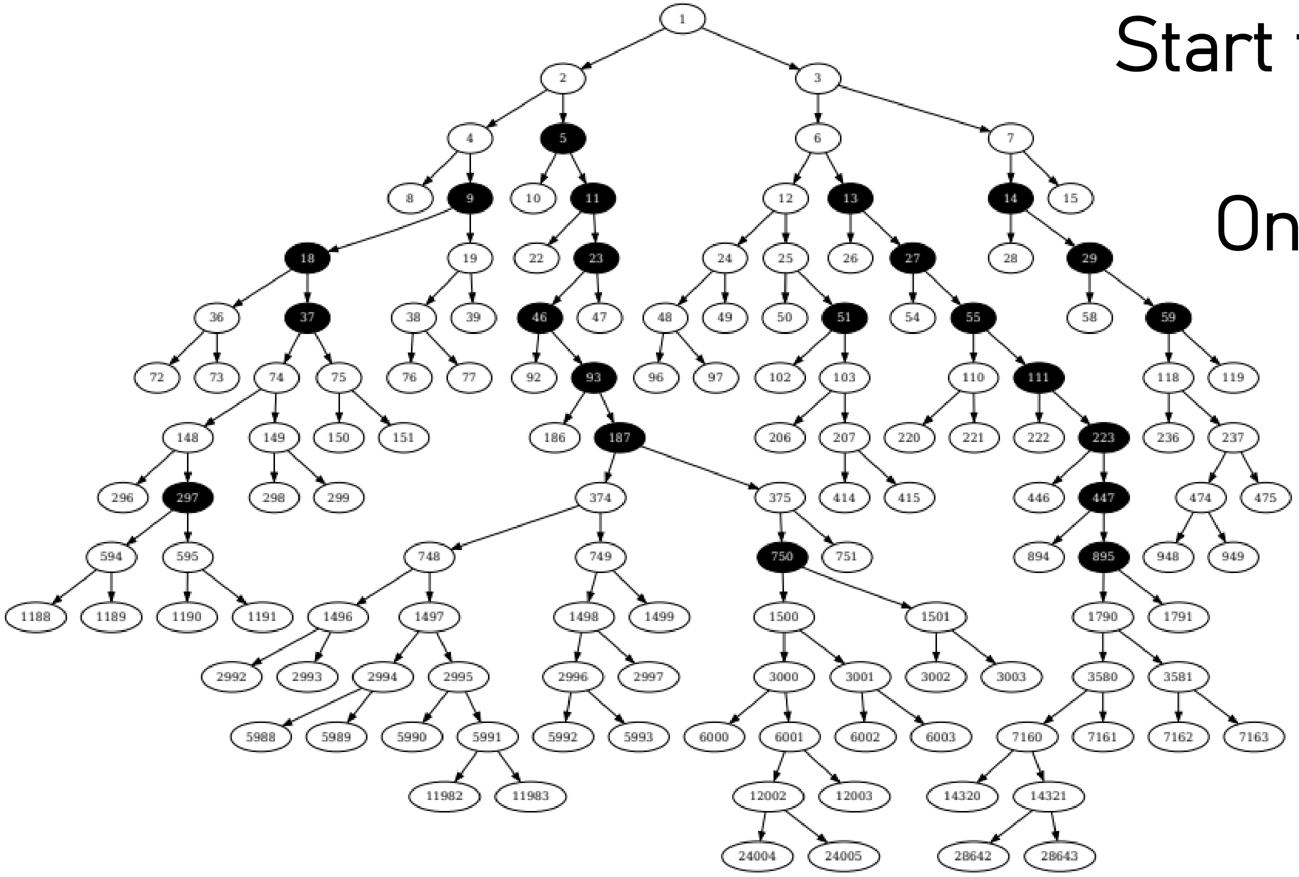
"Stochastic inflation" in INSPIRE-HEP

- 164 papers
- 98 of these since 2015



Stochastic trees

Animali:2025pyf



Start from 1 Hubble patch

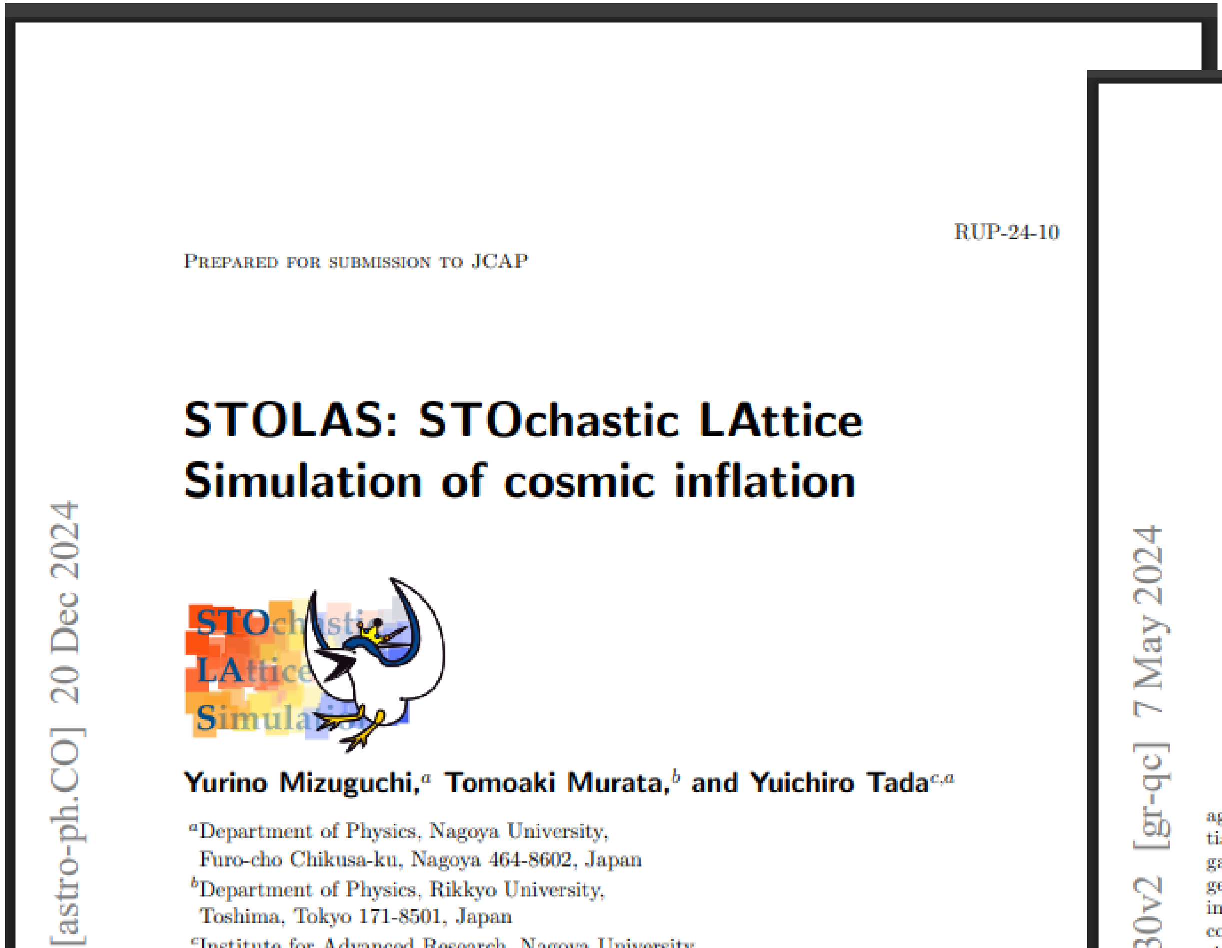
Once volume doubled: split in two

Spatial distribution?
Clustering?

Lattice simulations

Mizuguchi:2024kbl

Stochastic grid



[astro-ph.CO] 20 Dec 2024

RUP-24-10

Prepared for submission to JCAP

STOLAS: STOchastic LAttice Simulation of cosmic inflation

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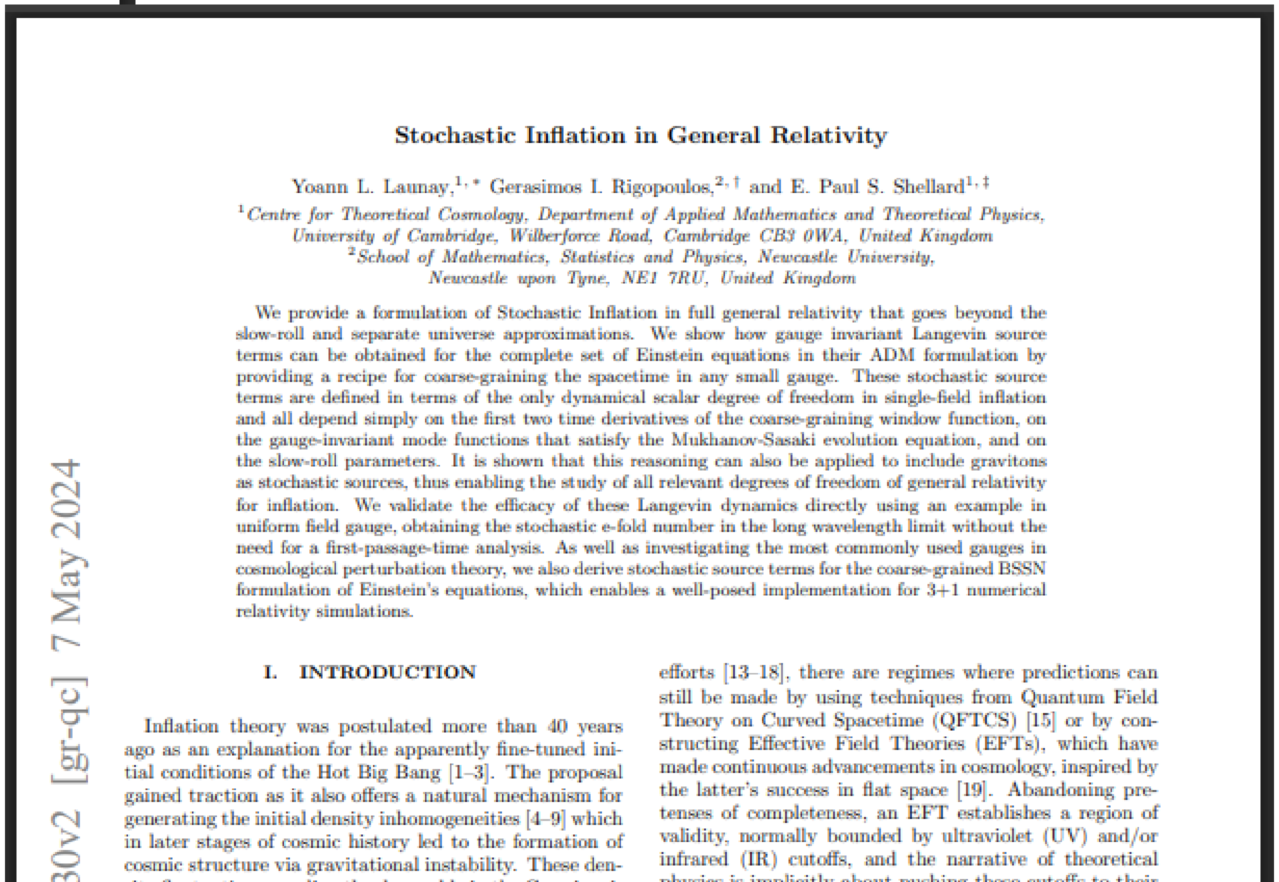
[b]Department of Physics, Rikkyo University, Toshima, Tokyo 171-8501, Japan

Launay:2024qsm

Combining stochastics and numerical relativity



7 May 2024

Stochastic Inflation in General Relativity

Yoann L. Launay,[1,*] Gerasimos I. Rigopoulos,[2,†] and E. Paul S. Shellard[1,‡]

[1]*Centre for Theoretical Cosmology, Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, United Kingdom*

[2]*School of Mathematics, Statistics and Physics, Newcastle University, Newcastle upon Tyne, NE1 7RU, United Kingdom*

We provide a formulation of Stochastic Inflation in full general relativity that goes beyond the slow-roll and separate universe approximations. We show how gauge invariant Langevin source terms can be obtained for the complete set of Einstein equations in their ADM formulation by providing a recipe for coarse-graining the spacetime in any small gauge. These stochastic source terms are defined in terms of the only dynamical scalar degree of freedom in single-field inflation and all depend simply on the first two time derivatives of the coarse-graining window function, on the gauge-invariant mode functions that satisfy the Mukhanov-Sasaki evolution equation, and on the slow-roll parameters. It is shown that this reasoning can also be applied to include gravitons as stochastic sources, thus enabling the study of all relevant degrees of freedom of general relativity for inflation. We validate the efficacy of these Langevin dynamics directly using an example in uniform field gauge, obtaining the stochastic e-fold number in the long wavelength limit without the need for a first-passage-time analysis. As well as investigating the most commonly used gauges in cosmological perturbation theory, we also derive stochastic source terms for the coarse-grained BSSN formulation of Einstein's equations, which enables a well-posed implementation for 3+1 numerical relativity simulations.

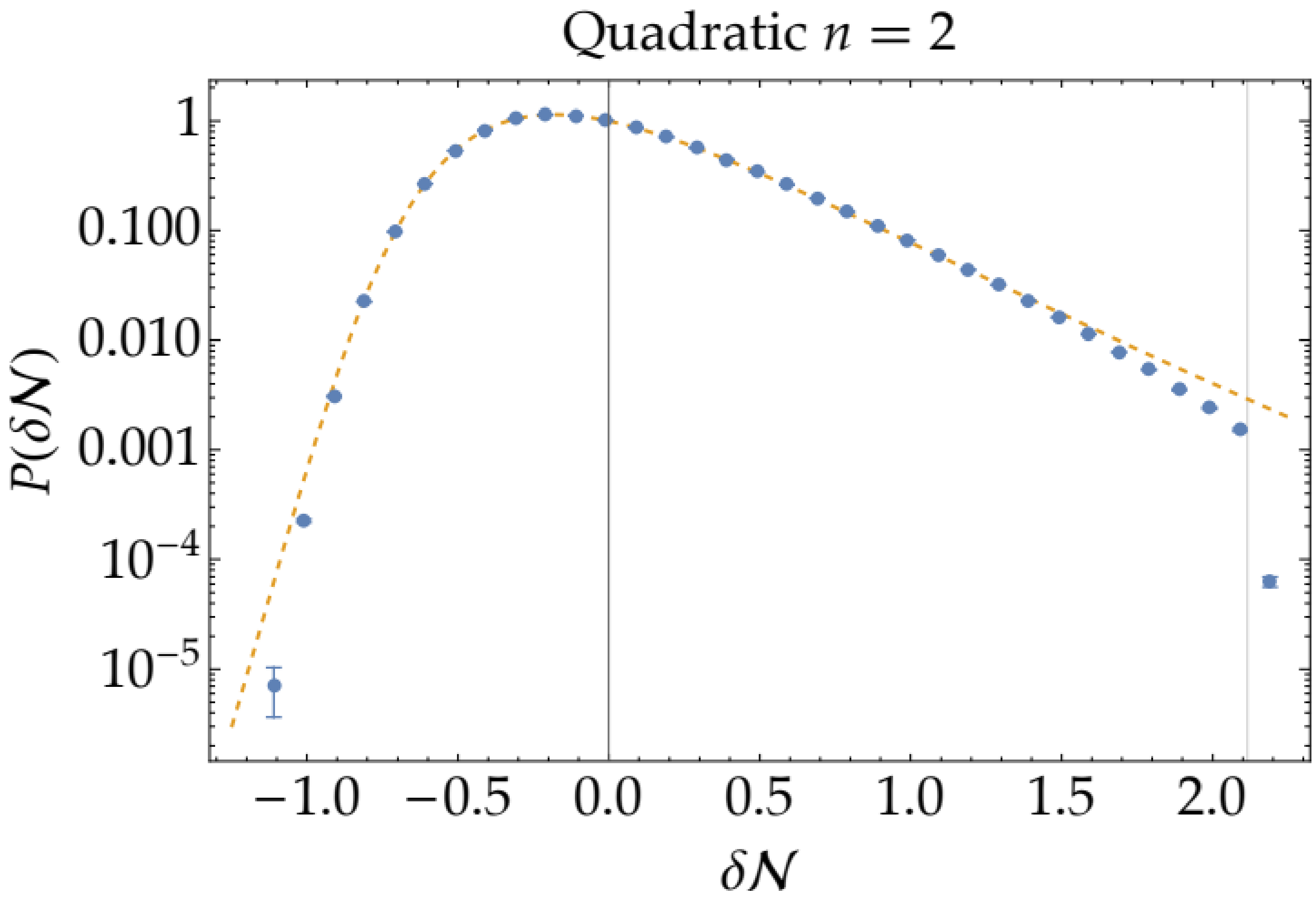
I. INTRODUCTION

Inflation theory was postulated more than 40 years ago as an explanation for the apparently fine-tuned initial conditions of the Hot Big Bang [1–3]. The proposal gained traction as it also offers a natural mechanism for generating the initial density inhomogeneities [4–9] which in later stages of cosmic history led to the formation of cosmic structure via gravitational instability. These den-

efforts [13–18], there are regimes where predictions can still be made by using techniques from Quantum Field Theory on Curved Spacetime (QFTCS) [15] or by constructing Effective Field Theories (EFTs), which have made continuous advancements in cosmology, inspired by the latter's success in flat space [19]. Abandoning pretenses of completeness, an EFT establishes a region of validity, normally bounded by ultraviolet (UV) and/or infrared (IR) cutoffs, and the narrative of theoretical

Multi-field inflation

Murata:2025onc



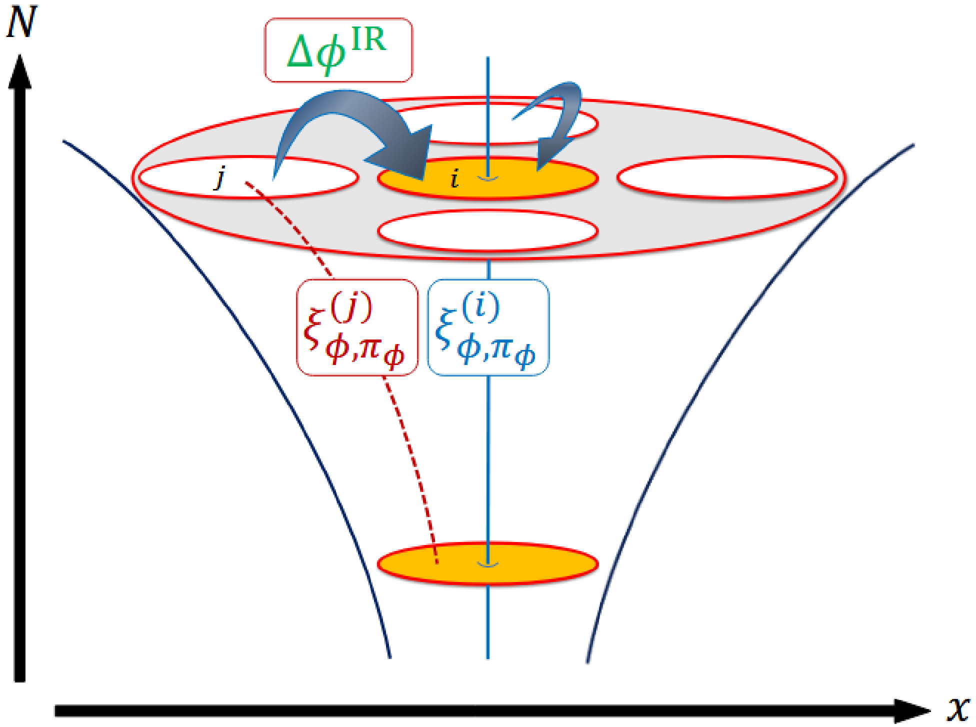
Hybrid inflation model

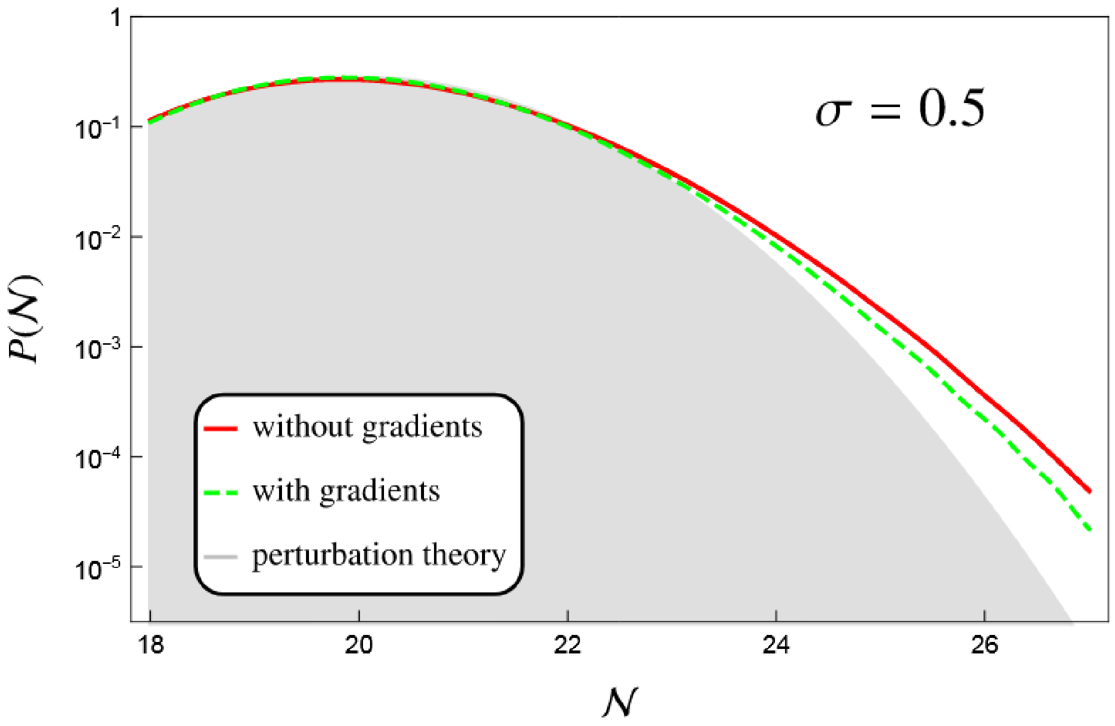
Upper bound to curvature perturbation

Gradient corrections

Briaud:2025ayt

Go beyond leading order in gradient expansion



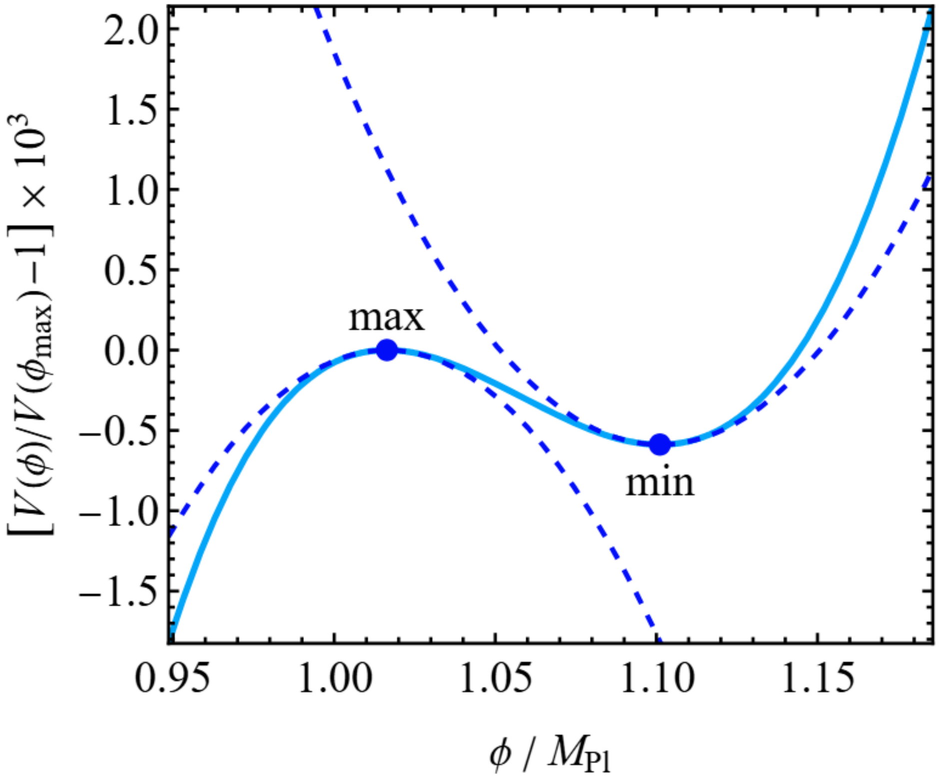


Colored noise
Exponential tails: a pullback effect

Eternal inflation

Tomberg:2025fku

$$\text{eternal inflation} \quad \Longleftrightarrow \quad \lim_{N\to\infty} \langle V \rangle_N \equiv \lim_{N\to\infty} \int_{\phi_{b1}}^{\phi_{b2}} e^{3N} P(\phi, N) \mathrm{d}\phi > 0$$



$$P(\phi, N) \sim e^{-\lambda N} \qquad \lambda \leq 3$$

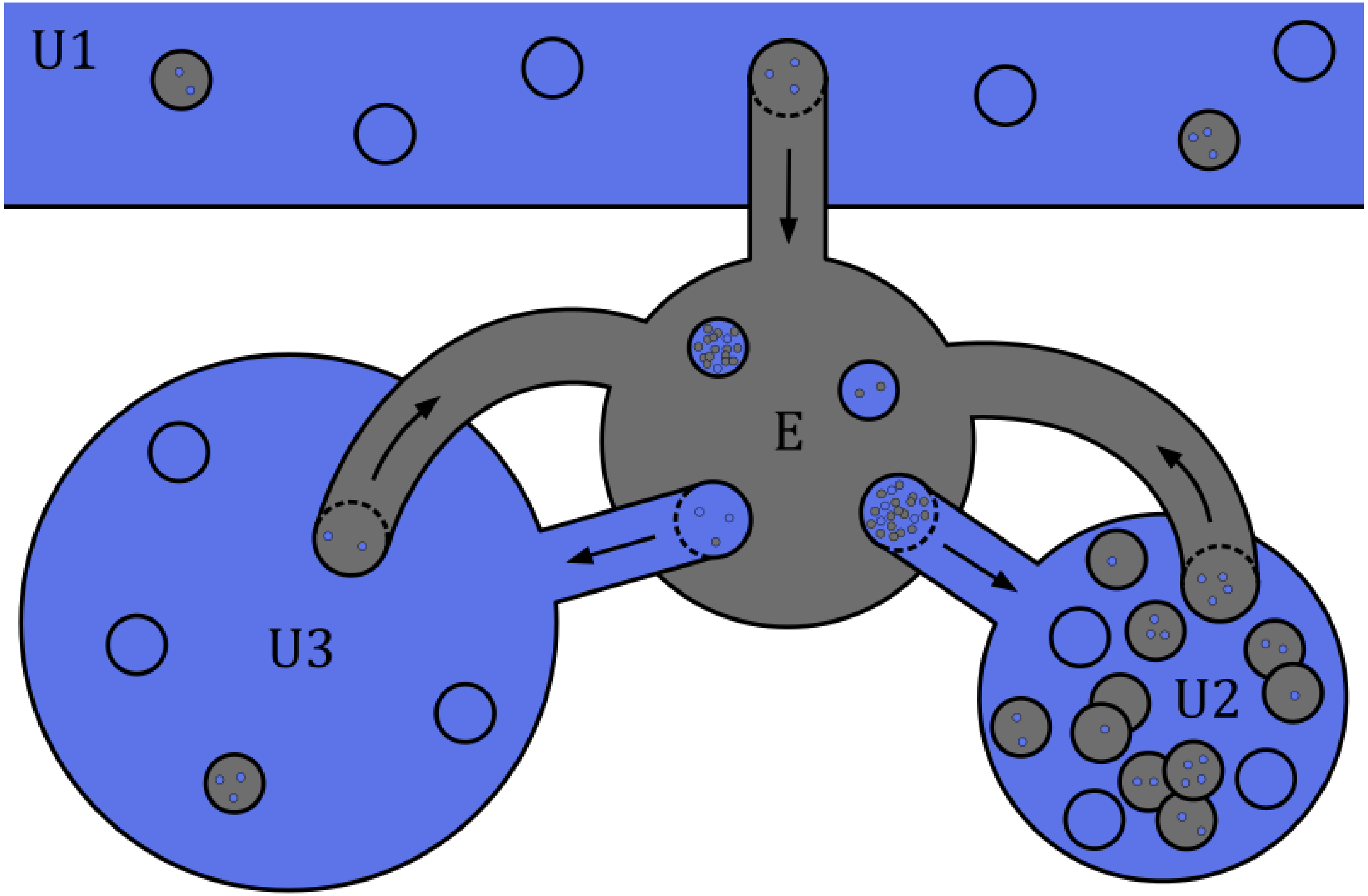
Maximum:

$$\lambda \approx |\eta_H| \sim 0.1$$

Minimum:

$$\lambda \approx 0$$

Eternal inflation



SUMMARY

Stochastic inflation gives a non-perturbative handle on inflationary perturbations

Useful for primordial black hole studies

Developments in various directions:
spiky profiles, stochastic trees, multi-field setups, gradient corrections…