Primordial black holes from stochastic constant-roll inflation

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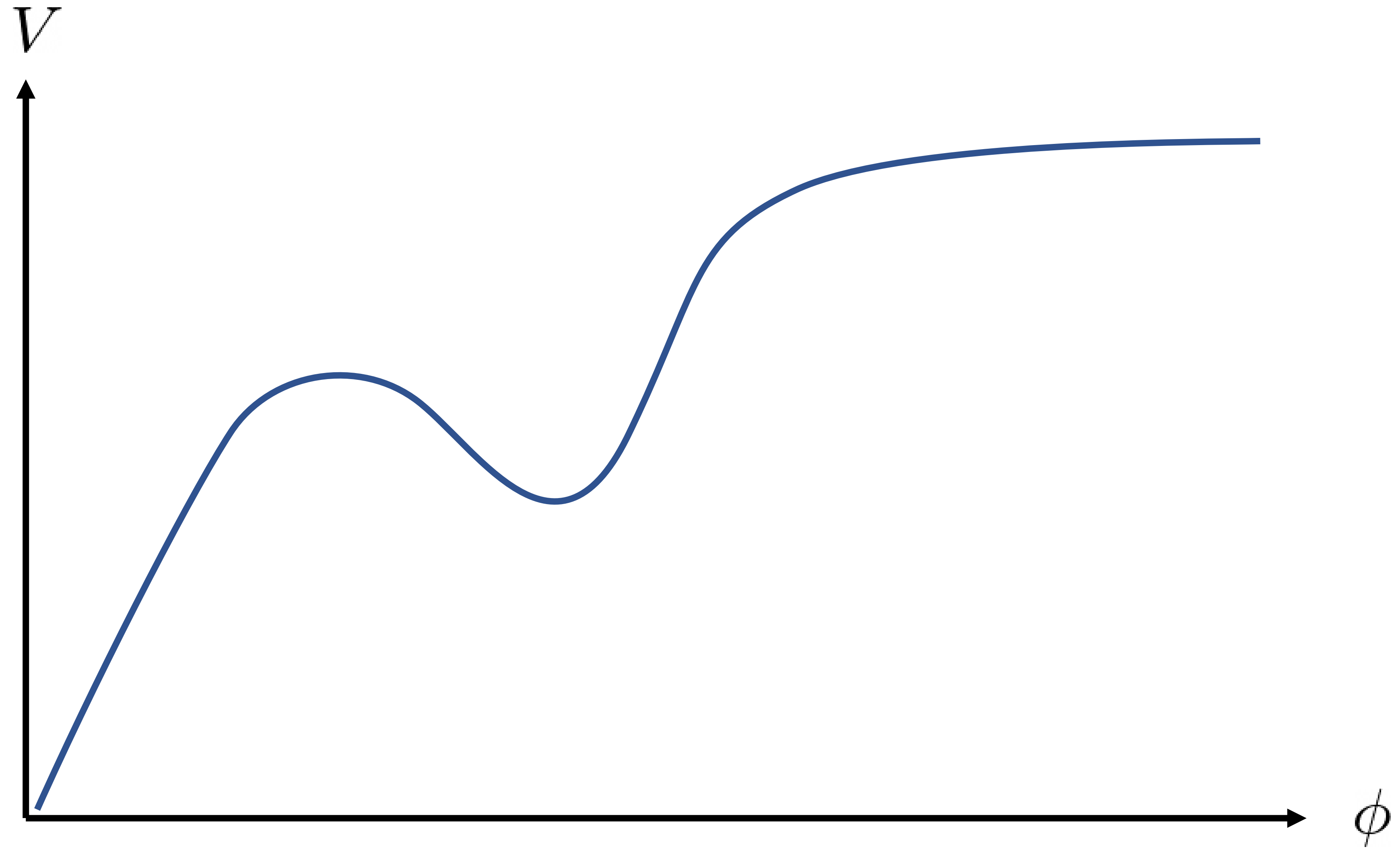
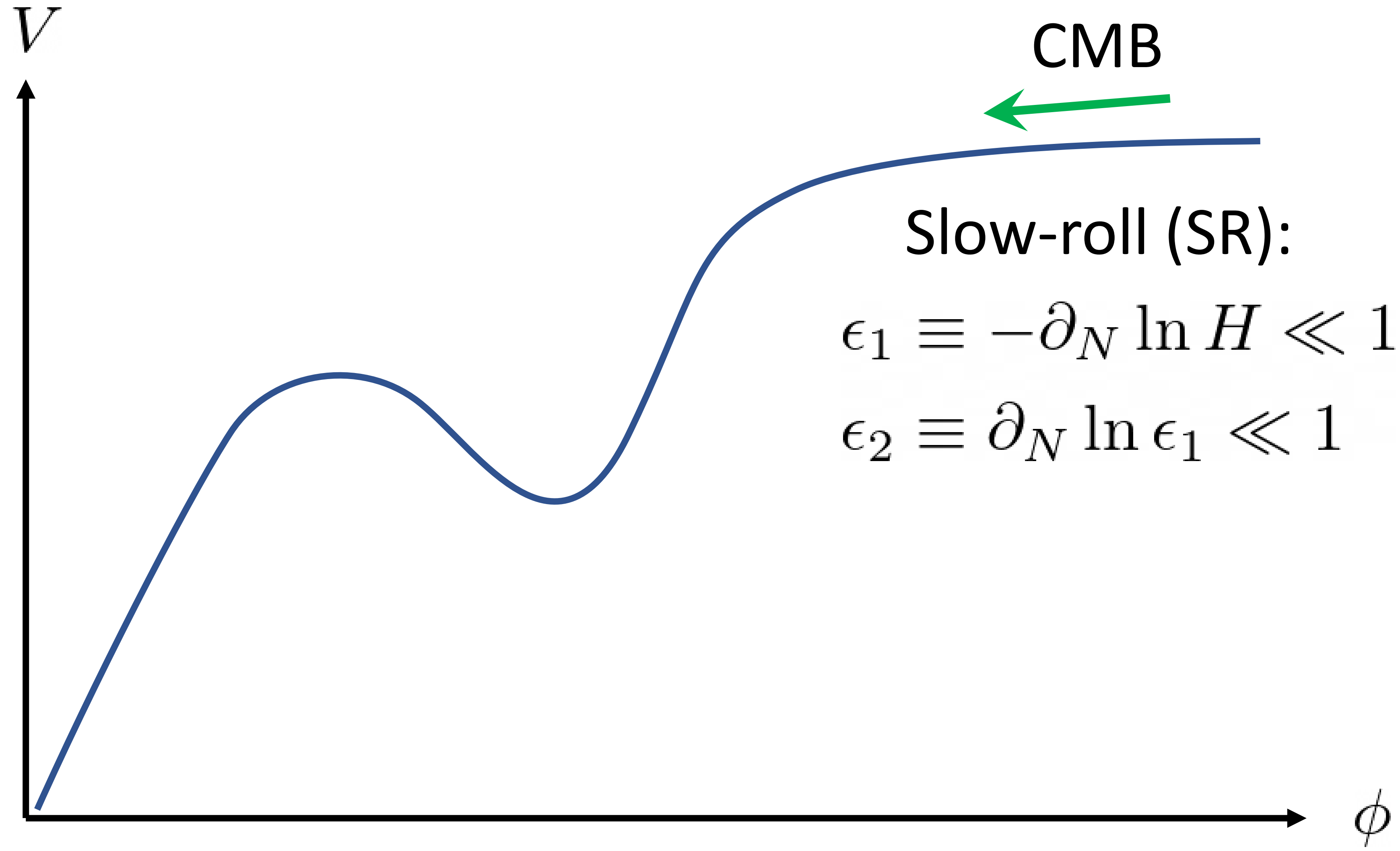
in collaboration with D. Figueroa, S. Raatikainen, S. Räsänen



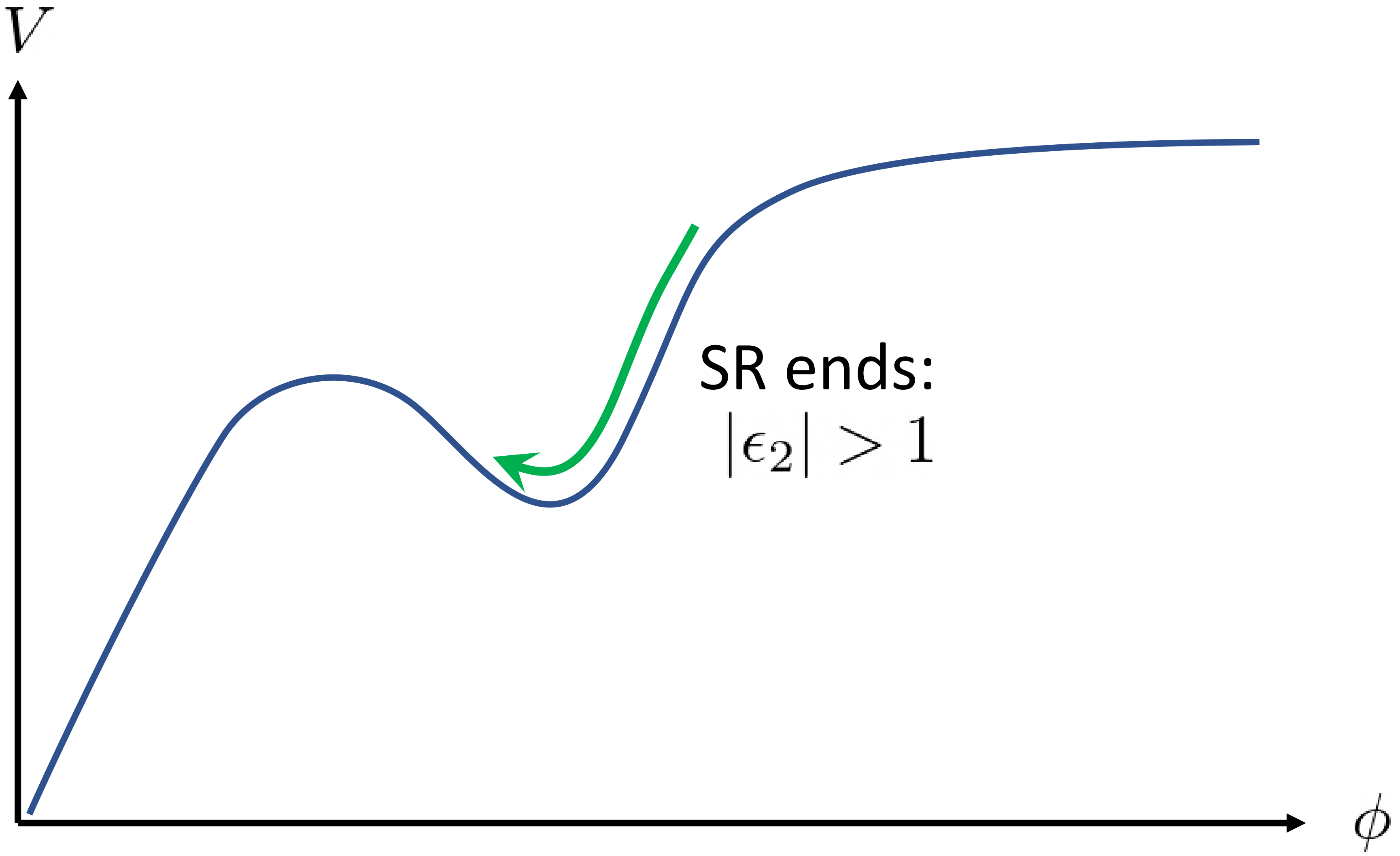
V
ϕ

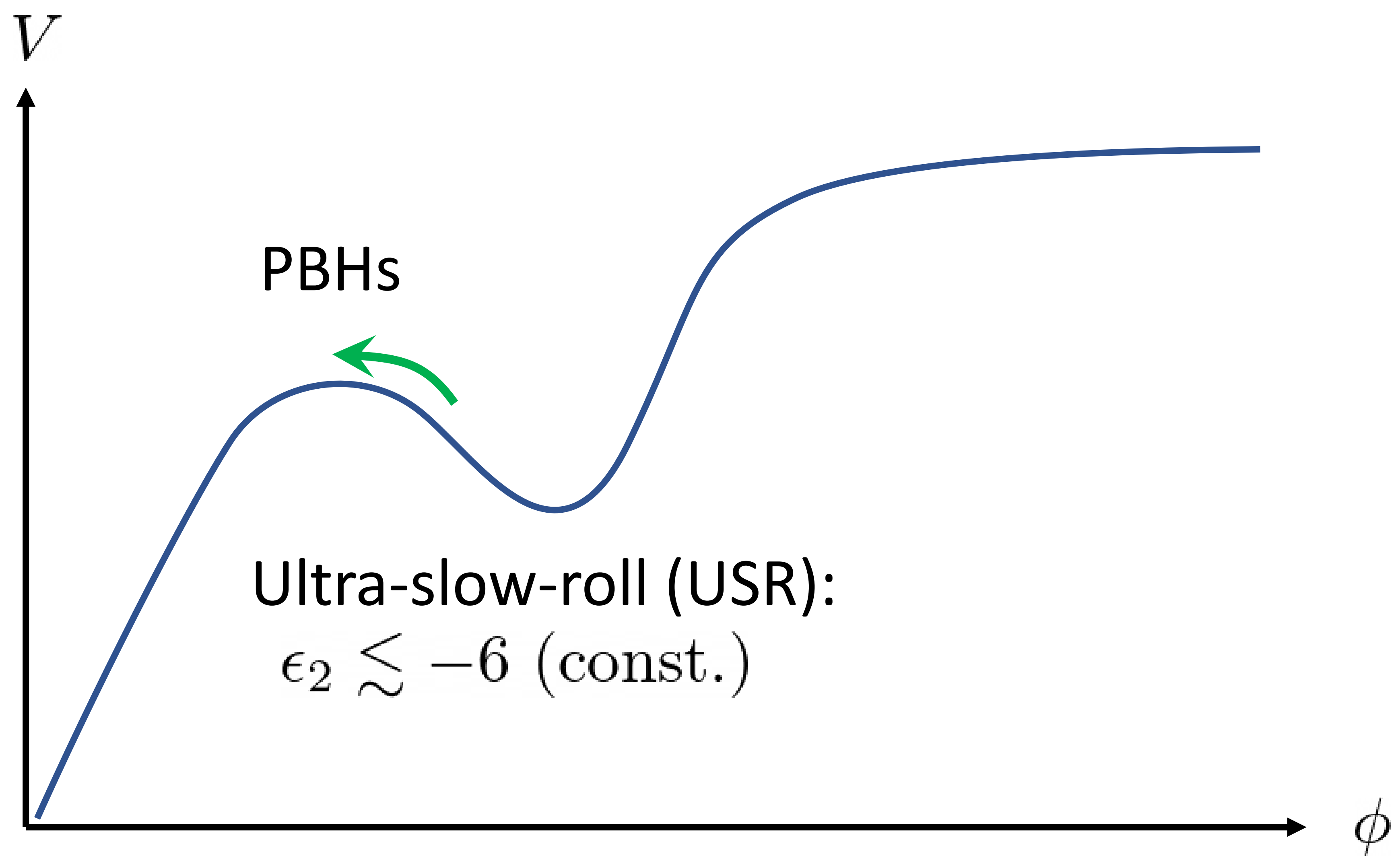


Slow-roll (SR):

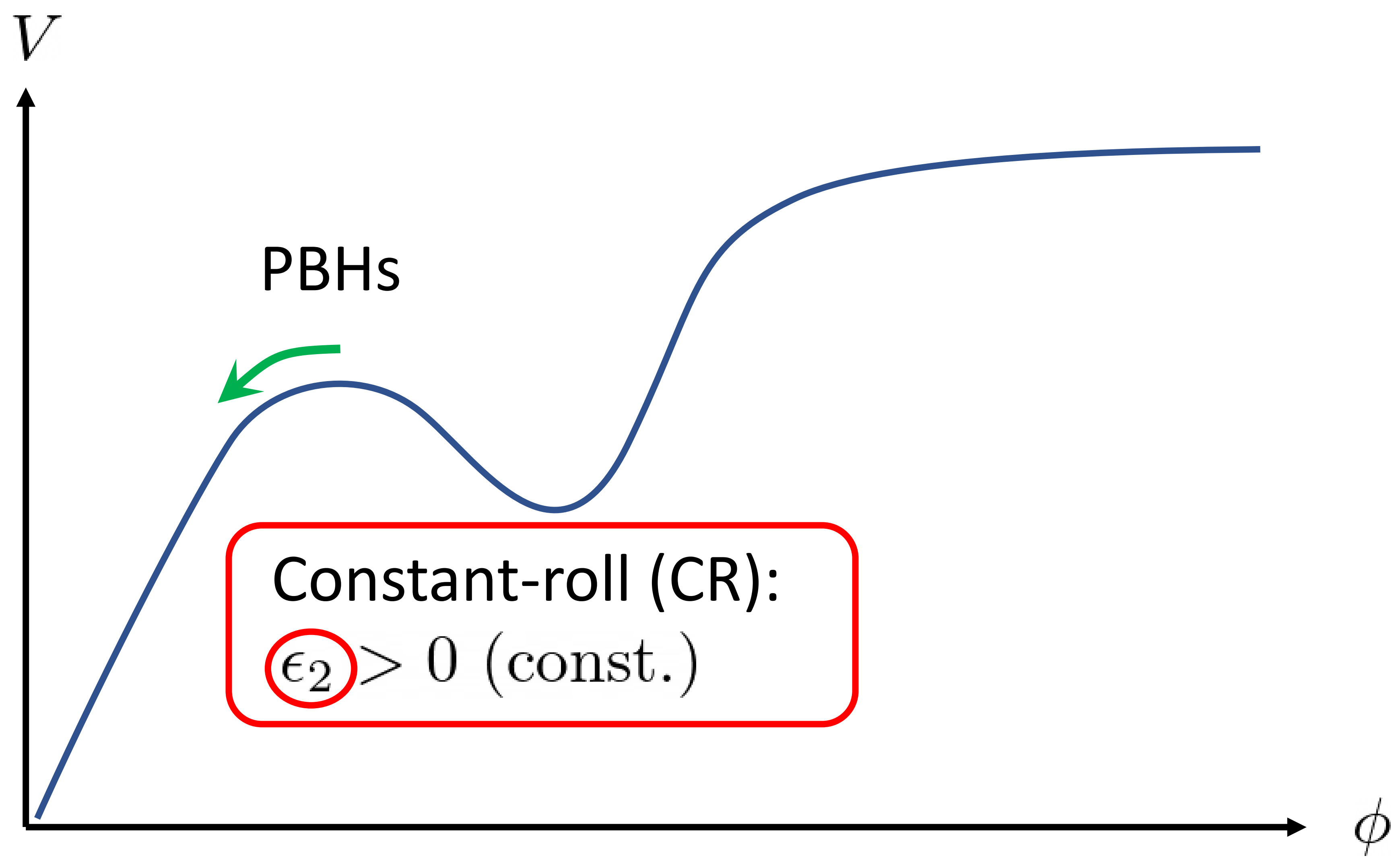
$$\epsilon_1 \equiv -\partial_N \ln H \ll 1$$

$$\epsilon_2 \equiv \partial_N \ln \epsilon_1 \ll 1$$

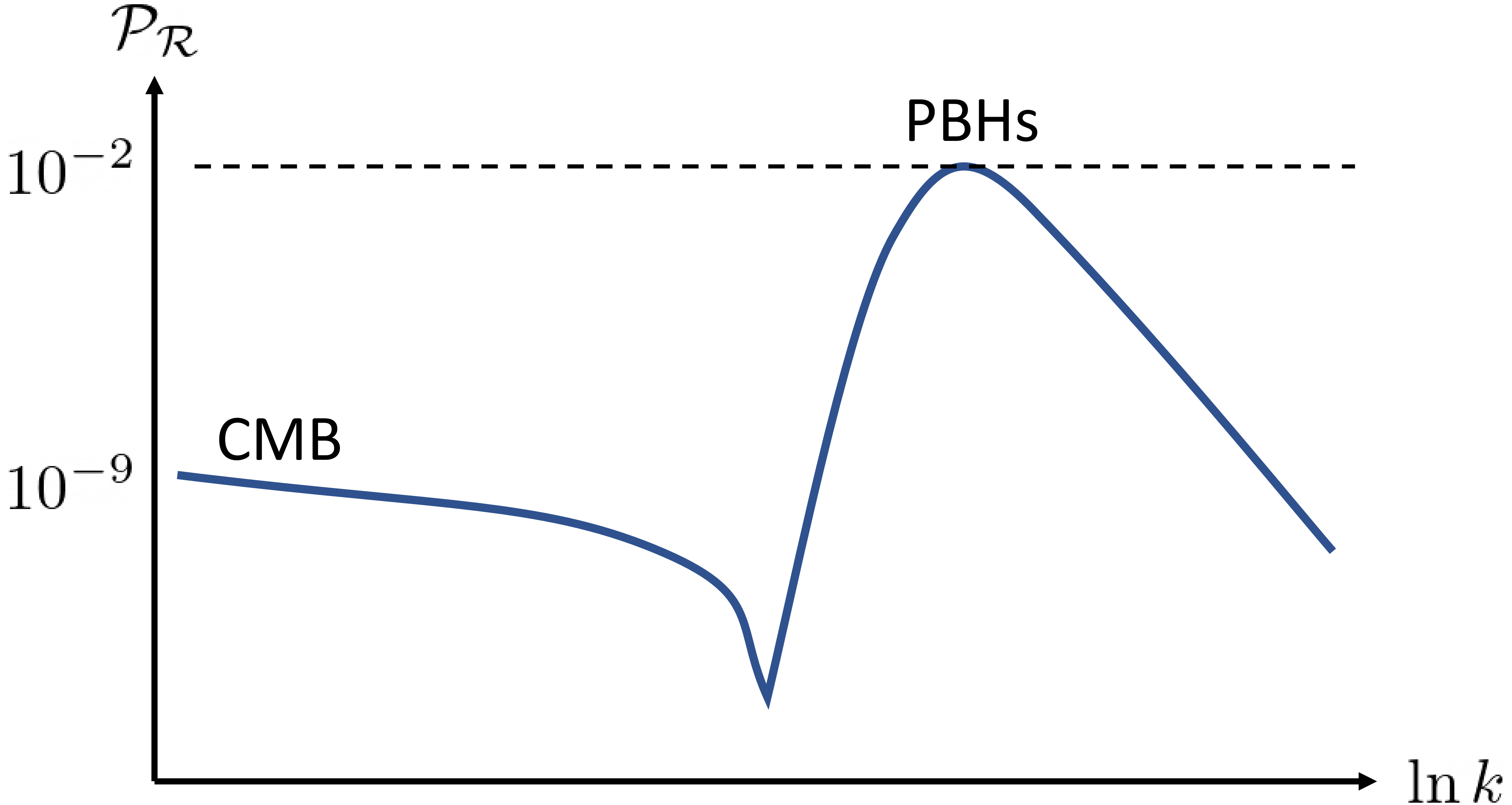

V
SR ends:
$|\epsilon_2| > 1$
ϕ



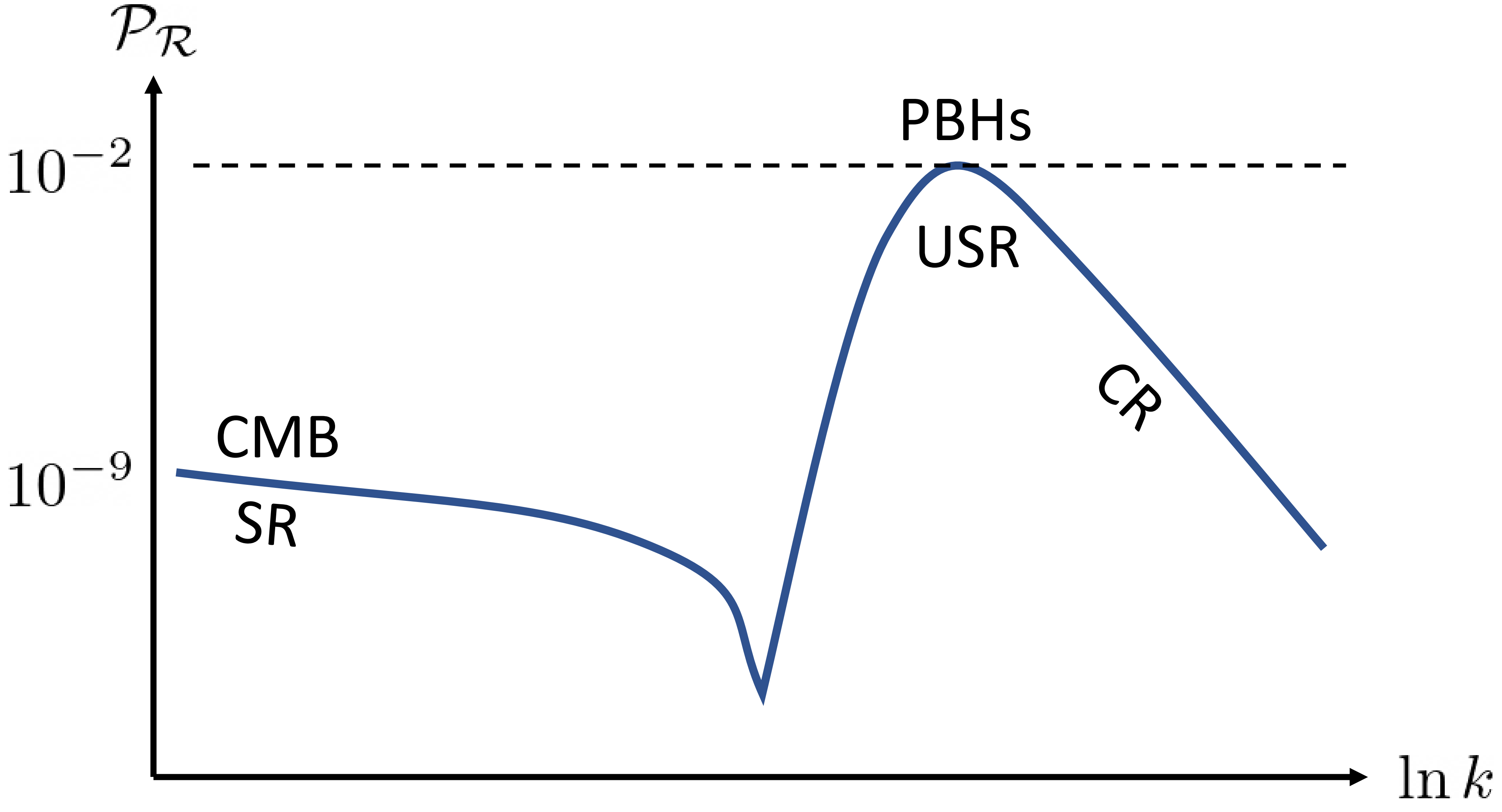
V
PBHs
Ultra-slow-roll (USR):
$\epsilon_2 \lesssim -6$ (const.)
ϕ

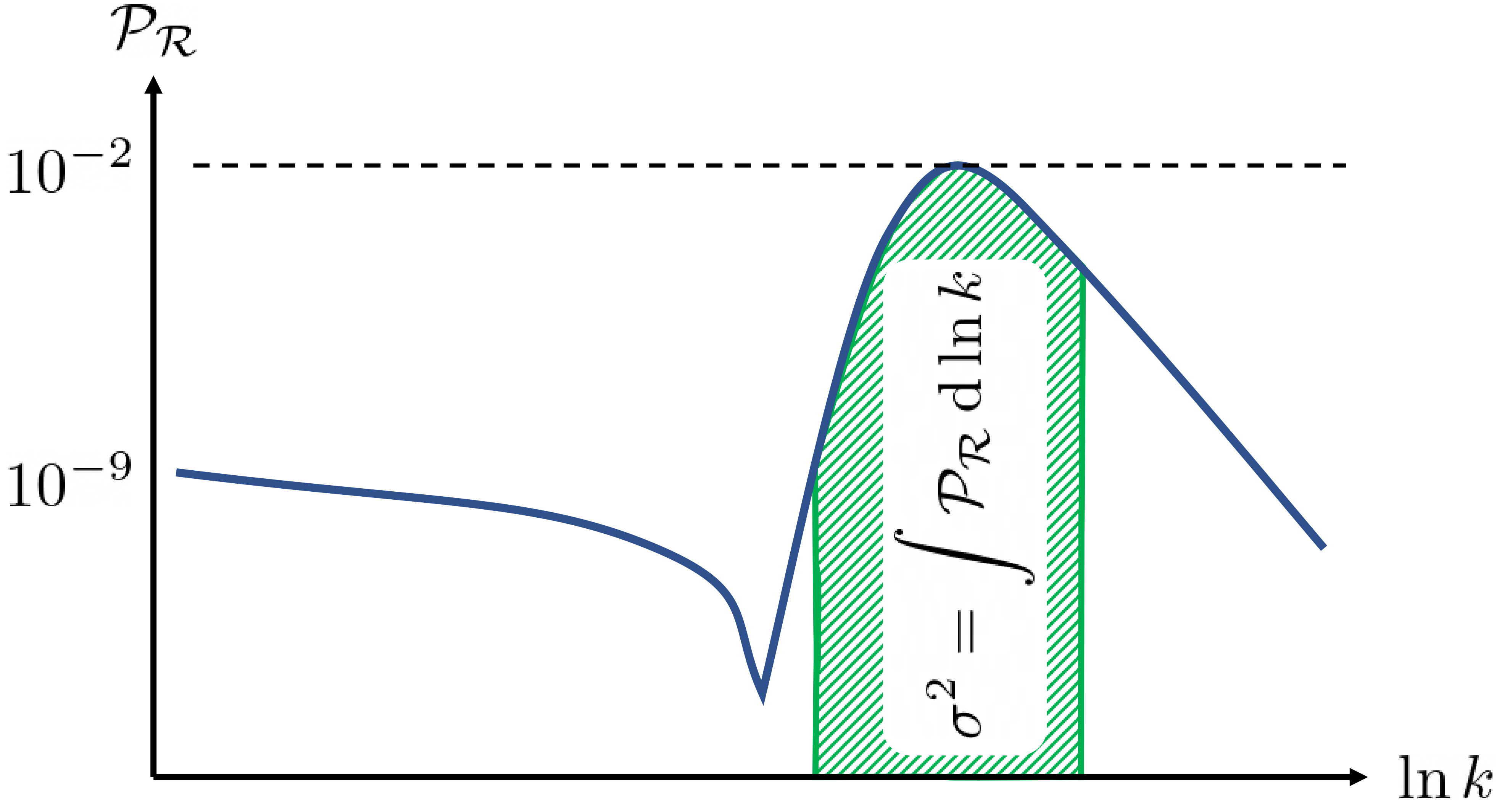


V
PBHs
Constant-roll (CR):
$\epsilon_2 > 0$ (const.)
ϕ



$\mathcal{P}_\mathcal{R}$
PBHs
10^{-2}
CMB
10^{-9}
$\ln k$


$\mathcal{P}_{\mathcal{R}}$
10^{-2}
10^{-9}
PBHs
USR
CR
CMB
SR
$\ln k$


$\mathcal{P}_\mathcal{R}$
10^{-2}
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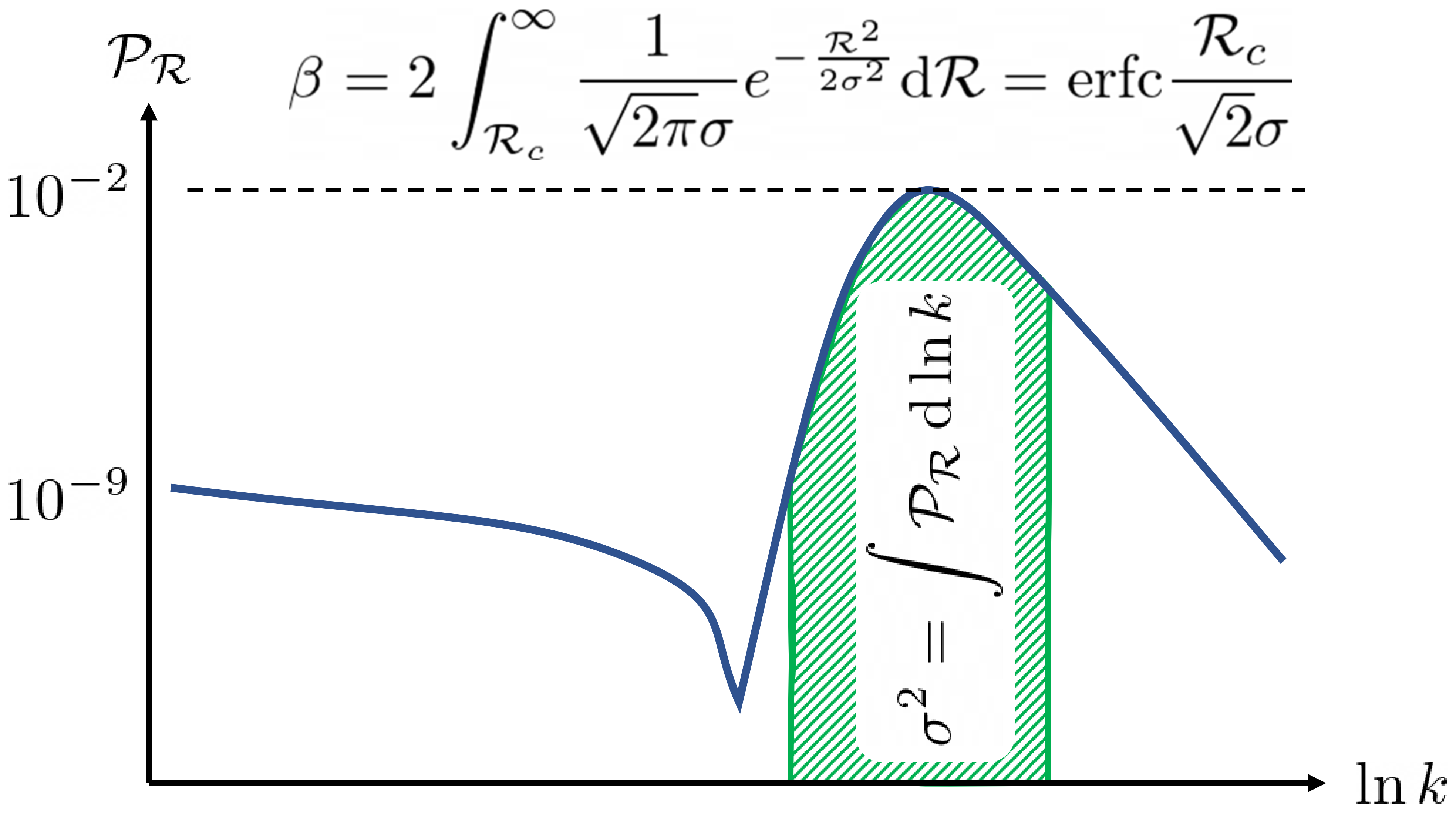
$\ln k$
$\sigma^2 = \int \mathcal{P}_\mathcal{R} \, \mathrm{d}\ln k$



$\mathcal{P}_{\mathcal{R}}$
$\beta = 2\int_{\mathcal{R}_c}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{\mathcal{R}^2}{2\sigma^2}} \mathrm{d}\mathcal{R} = \mathrm{erfc}\frac{\mathcal{R}_c}{\sqrt{2}\sigma}$
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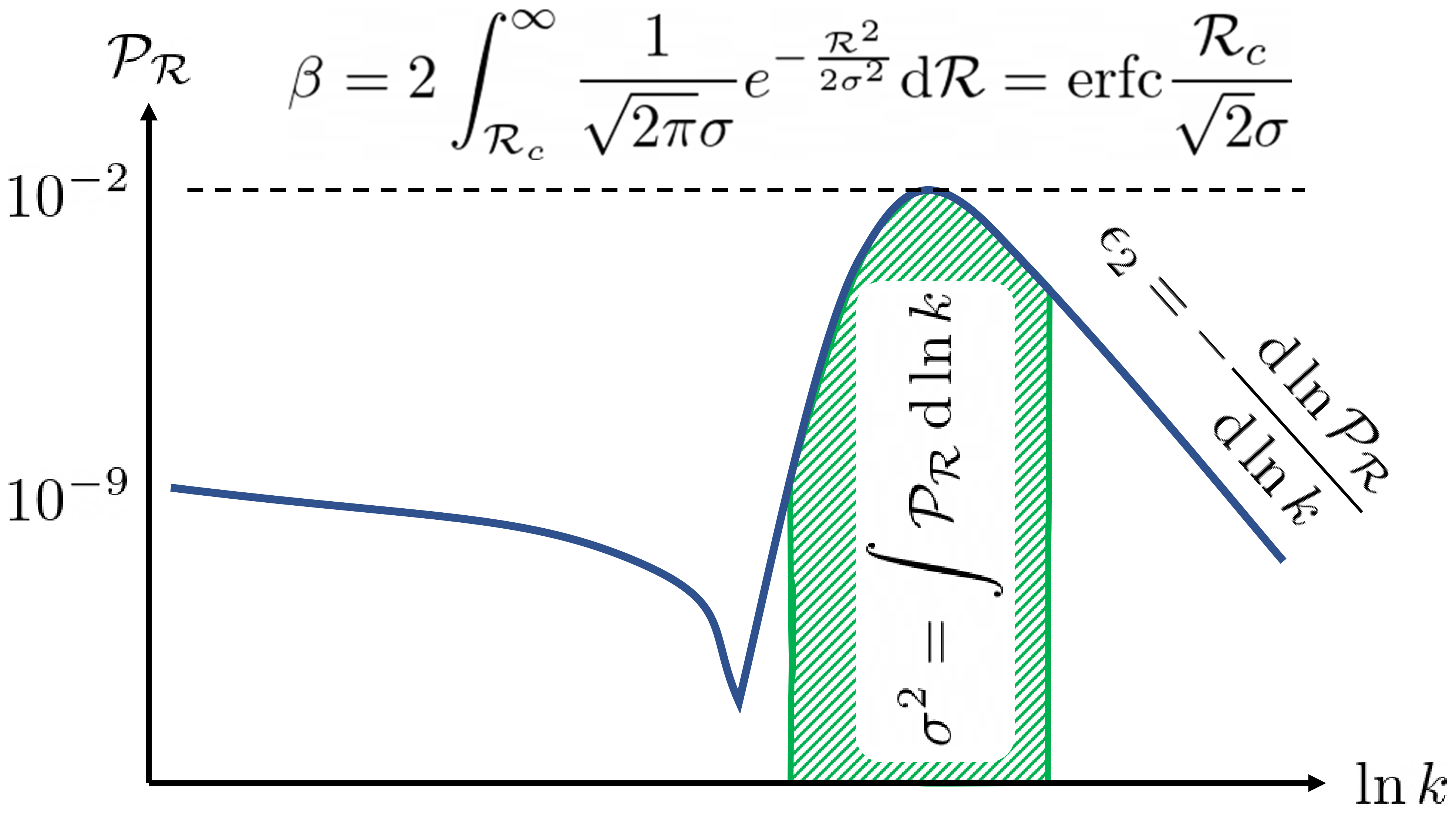

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Example case of dark matter:

$$\beta \approx 10^{-17} \Rightarrow \mathcal{R}_c \approx 8.5\sigma$$

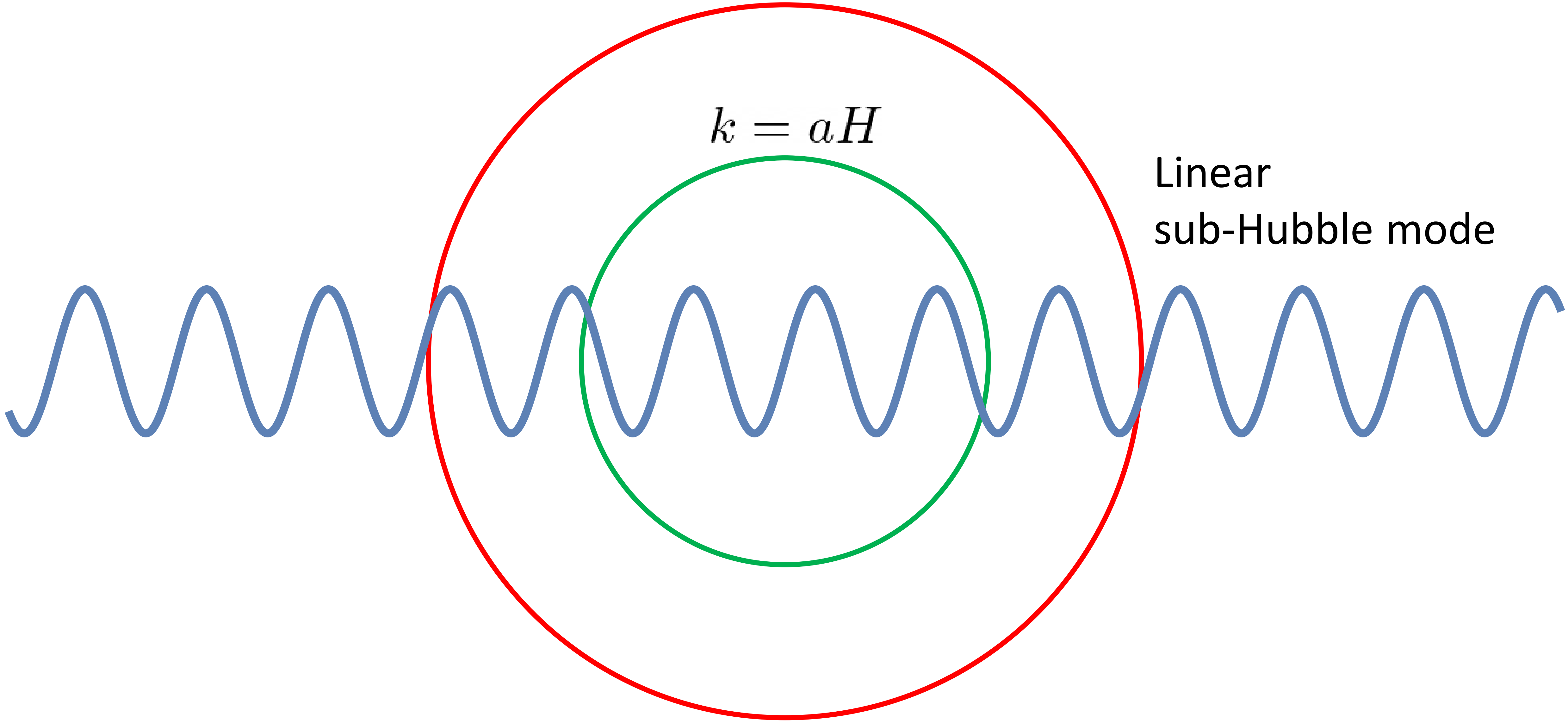
Non-Gaussianities important?

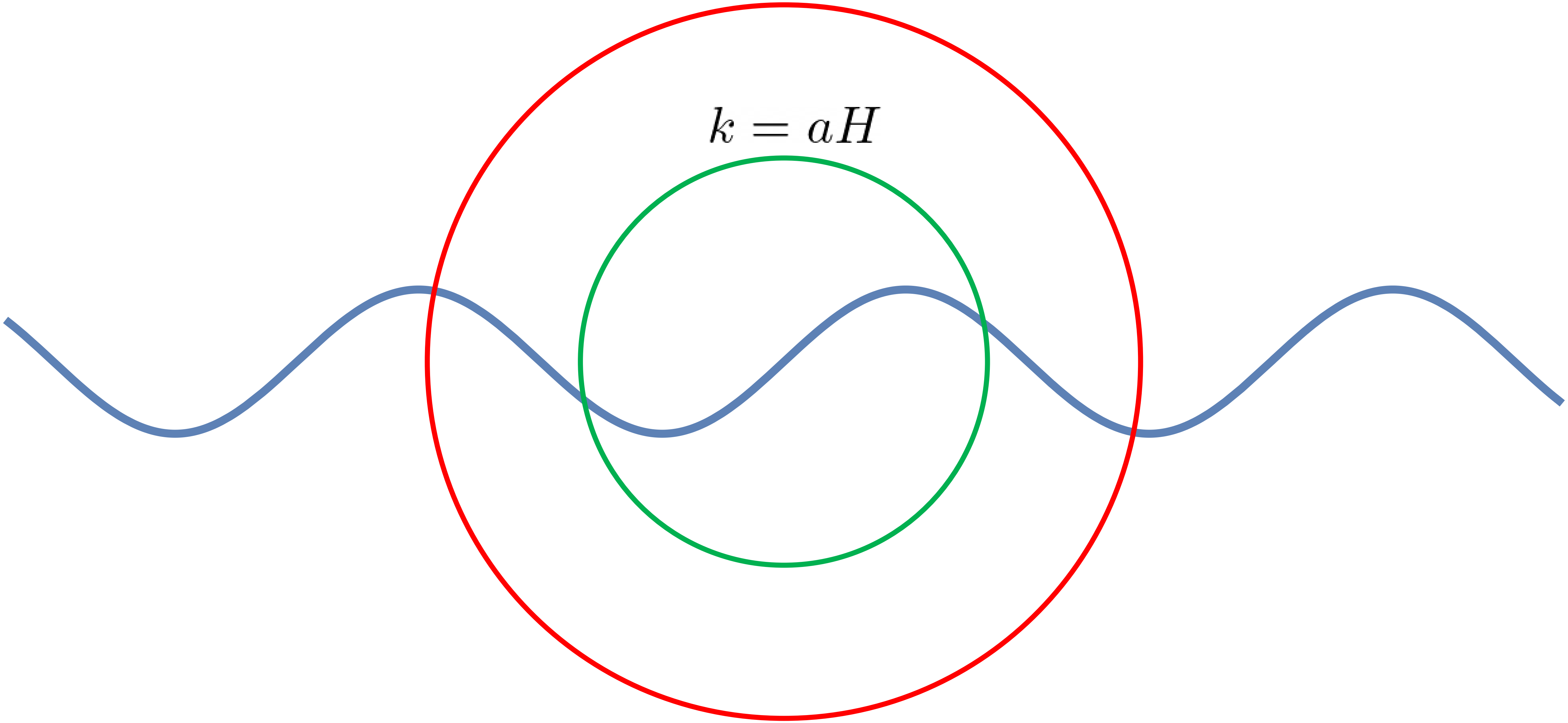
Stochastic inflation

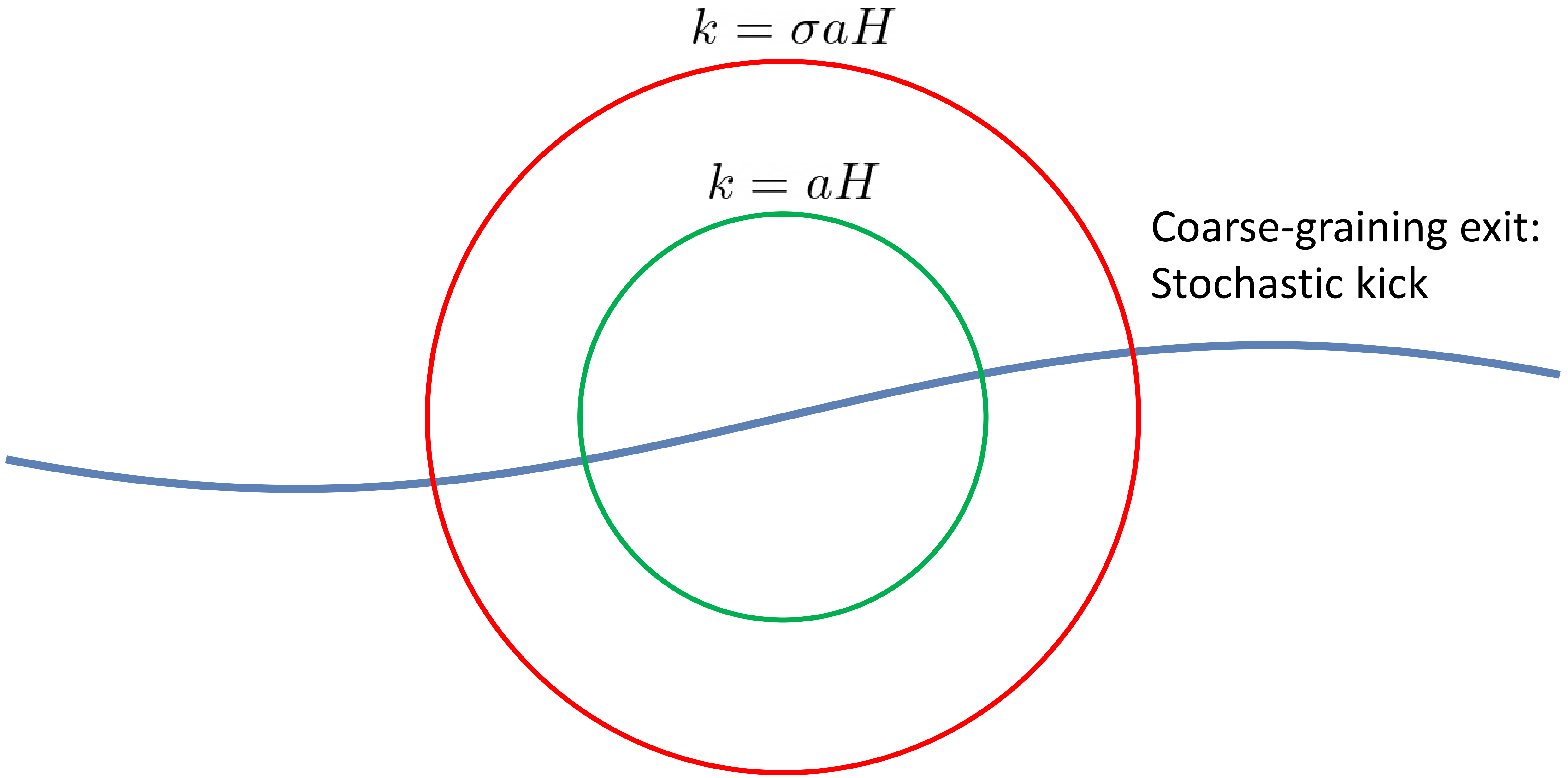
$$\phi' = \pi + \xi_\phi\,, \quad \pi' = -(3-\epsilon_1)\left(\pi + \frac{V_{,\phi}(\phi)}{V(\phi)}\right) + \xi_\pi\,,$$

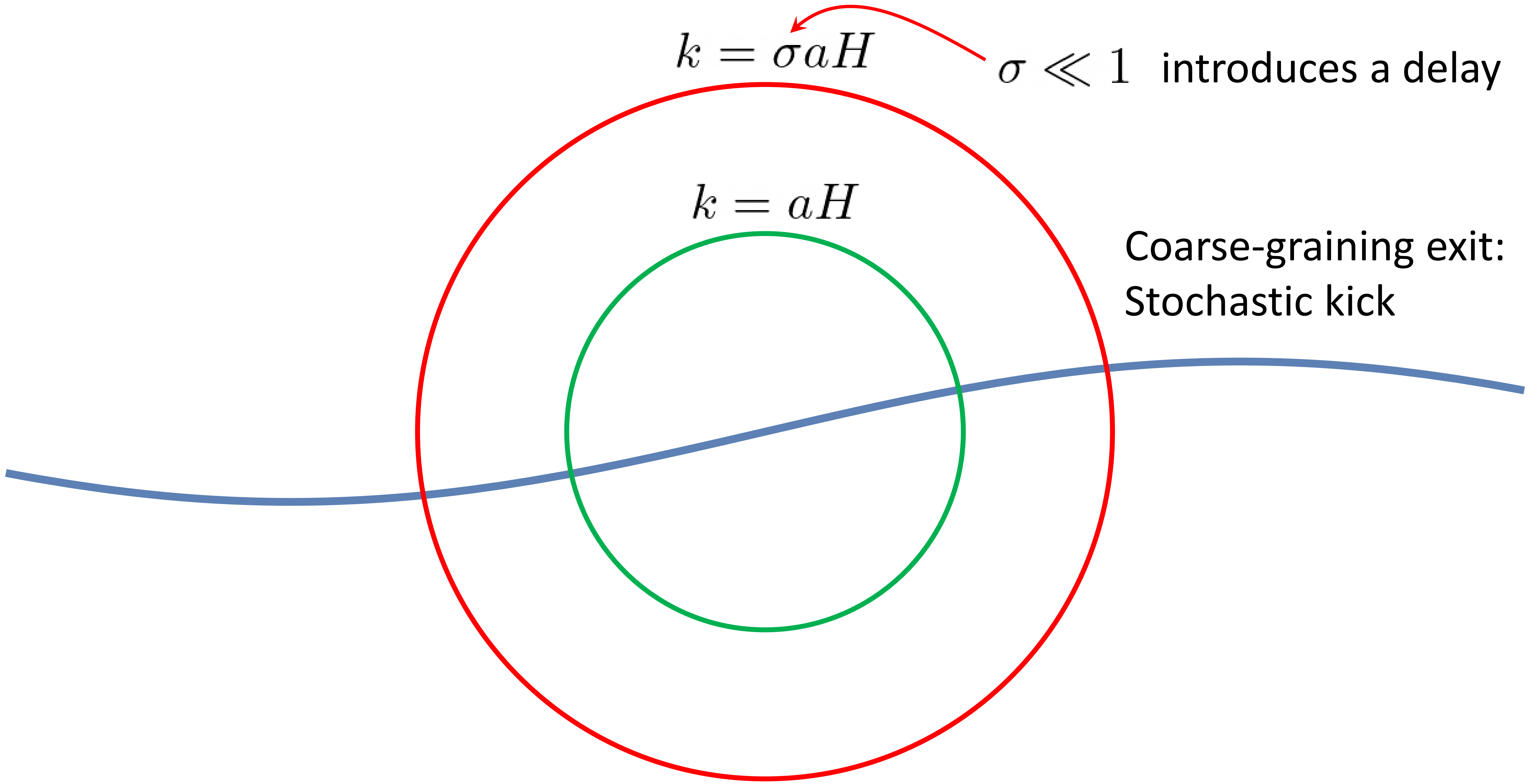
$$\langle \xi_\phi(N)\xi_\phi(N')\rangle = \frac{k^3}{2\pi^2}|\delta\phi_{k=\sigma aH}(N)|^2\delta(N-N')$$

ΔN formalism: $\mathcal{R} = \Delta N \equiv N - \bar{N}$


$k = aH$
Linear
sub-Hubble mode


$k = aH$


$k = \sigma a H$
$k = aH$
Coarse-graining exit:
Stochastic kick


$k = \sigma a H$
$\sigma \ll 1$ introduces a delay
$k = aH$
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Stochastic kick

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…perturbations don't depend on stochasticity:

$\epsilon_0 \exp(\epsilon_2 N)$

$$\frac{k^3}{2\pi^3}|\delta\phi_k|^2 = \epsilon_1(N)\mathcal{P}_\mathcal{R}(k)$$

Pre-computed

Simplified stochastic equation:

$$\mathrm{d}\phi = \pi(\phi)\,\mathrm{d}N + \sqrt{2\epsilon_1(N)\mathcal{P}_{\mathcal{R}}(k=\sigma a H)}\,\hat{\xi}_N$$

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$$= \frac{\epsilon_2}{2}\phi$$

$$= \epsilon_0 \exp(\epsilon_2 N)$$

Simplified stochastic equation:

$$\mathrm{d}\phi = \pi(\phi)\,\mathrm{d}N + \sqrt{2\epsilon_1(N)\mathcal{P}_\mathcal{R}(k=\sigma a H)}\,\hat{\xi}_N \longleftarrow \langle \hat{\xi}_N \hat{\xi}_{N'} \rangle = \delta_{NN'}$$

$$\pi(\phi) = \frac{\epsilon_2}{2}\phi \qquad \epsilon_1(N) = \epsilon_0 \exp(\epsilon_2 N)$$

$$\phi(N) = \phi_0 \exp\left(\frac{\epsilon_2}{2}N\right)\left[1 - \frac{\epsilon_2}{2}X(N)\right],$$

$$X(N) \equiv -\sum_{k=k_{\mathrm{ini}}}^{k=\sigma a H} \sqrt{\mathcal{P}_\mathcal{R}(k)\,\mathrm{d}\ln k}\,\hat{\xi}_k = \frac{2}{\epsilon_2}\left(1 - e^{-\frac{\epsilon_2}{2}\Delta N}\right)$$

PBH abundance

$$p(X) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{X^2}{2\sigma^2}} \,, \qquad \sigma^2 \equiv \int_{k_{\mathrm{ini}}}^{k_{\mathrm{end}}} \mathcal{P}_{\mathcal{R}}(k) \, \mathrm{d} \ln k$$

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$$p(\Delta N) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{2}{\sigma^2\epsilon_2^2}\left(1 - e^{-\frac{\epsilon_2}{2}\Delta N}\right)^2 - \frac{\epsilon_2}{2}\Delta N\right]$$

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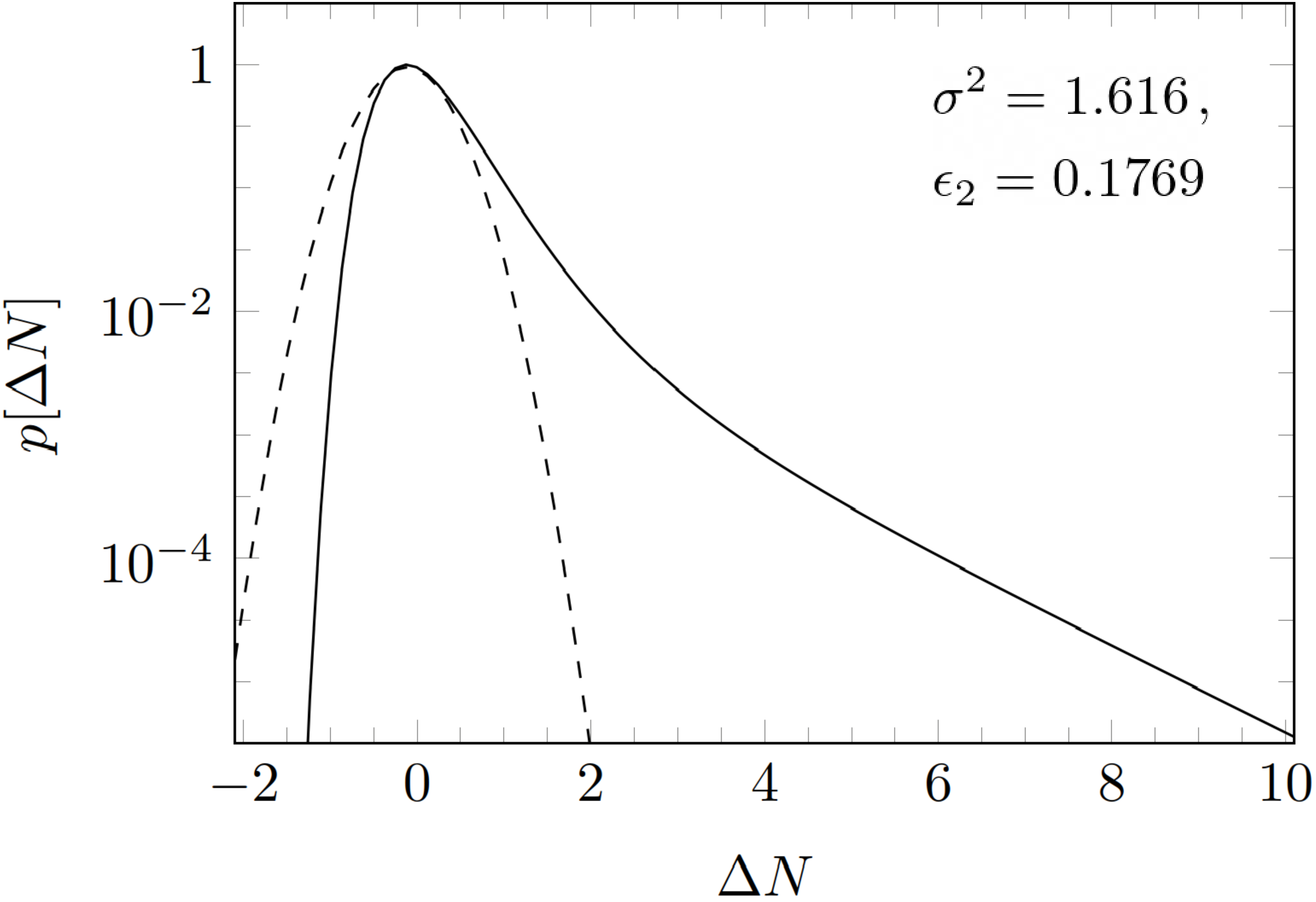
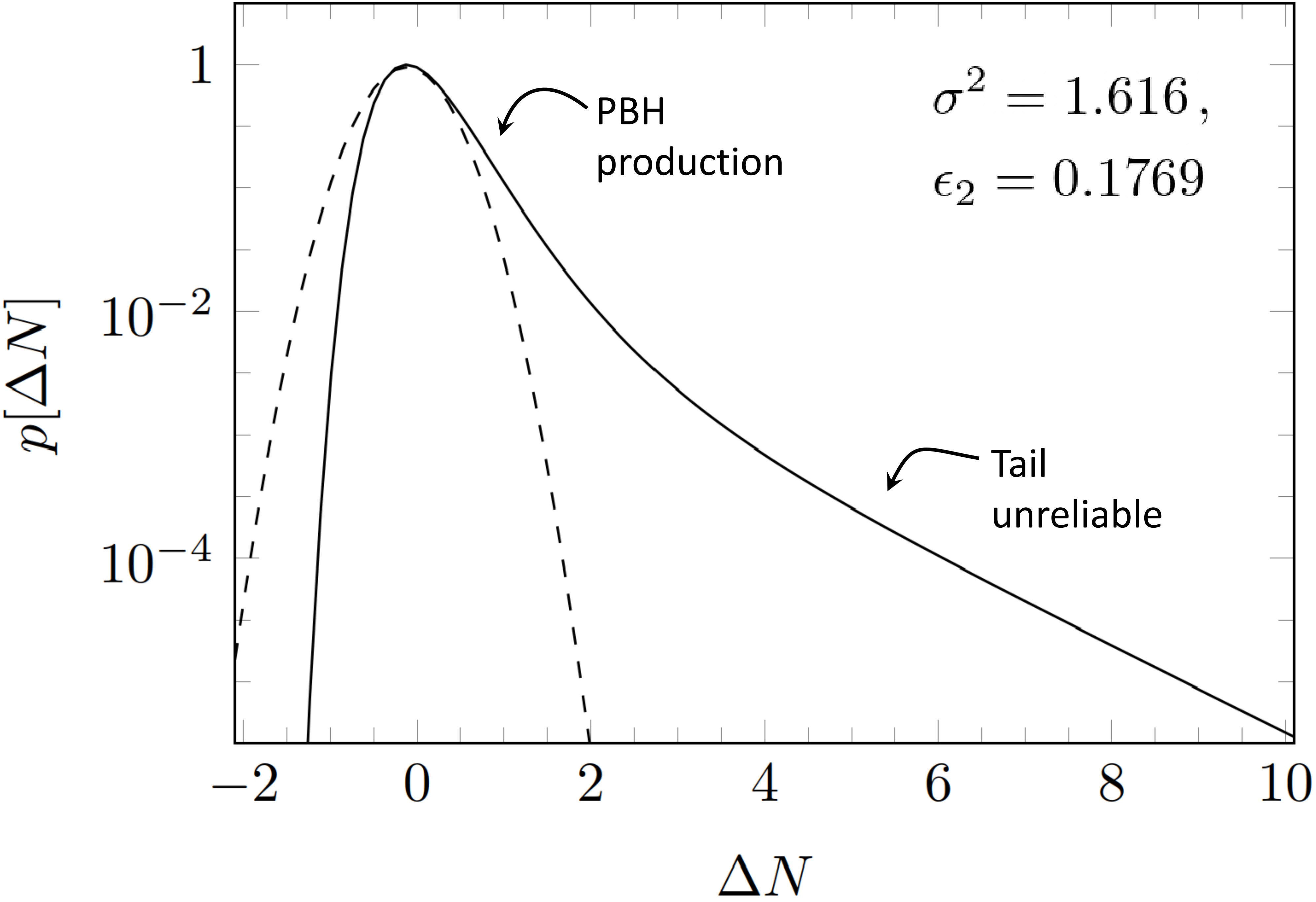
$$\beta = \mathrm{erfc}\left\{ \frac{\sqrt{2}}{\sigma \epsilon_2} \left[1 - \exp\left(-\frac{\epsilon_2}{2} \mathcal{R}_c \right) \right] \right\}$$



$\sigma^2 = 1.616\,,$
$\epsilon_2 = 0.1769$
$p[\Delta N]$
1
10^{-2}
10^{-4}
−2
0
2
4
6
8
10
ΔN



PBH
production
$\sigma^2 = 1.616\,,$
$\epsilon_2 = 0.1769$
Tail
unreliable
1
10^{-2}
10^{-4}
$p[\Delta N]$
−2
0
2
4
6
8
10
ΔN

For $\beta = 10^{-17}\,, \quad \mathcal{R}_c = 1$

Gaussian case: $\epsilon_2 = 0\,, \quad \sigma^2 = 0.014$

Maximal non-Gaussianity: $\epsilon_2 = 4.5\,, \quad \sigma^2 = 0.0022$

Future directions

Compaction-function based approach

Correlations between scales? PBH clustering?

Conclusions

Stochastic inflation introduces non-Gaussian corrections to PBH statistics

Constant-roll inflation is at the heart of PBH inflation models

Perturbation distribution easy to compute:
a function of σ^2 and ϵ_2