Preheating in Palatini Higgs inflation

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[1902.10148]

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Contents

- Higgs inflation: metric vs Palatini formalisms
- Reheating
- Significance on cosmology

Motivation

- Planck: spectral index $n_s = 0.9625 \pm 0.0048$ [1807.06211]
- Typical inflationary models: $n_s \approx 1 - aN^b$, $N \sim 50$ is number of e-folds of inflation

$$\Delta n_s \approx -abN^{b-1}\Delta N \approx b(n_s - 1)\frac{\Delta N}{N} \sim 10^{-3}\Delta N$$

General relativity: degrees of freedom

- Metric $g_{\mu\nu}$ gives local lengths and angles
- Connection $\Gamma^{\alpha}_{\beta\gamma}$ gives covariant derivatives
- Metric formulation: Levi-Civita connection

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\rho}(g_{\beta\rho,\gamma} + g_{\gamma\rho,\beta} - g_{\beta\gamma,\rho})$$

- Palatini formulation: $g_{\mu\nu}$ and $\Gamma^{\alpha}_{\beta\gamma}$ independent

Higgs inflation

- Non-minimal coupling to gravity

$$S = \int d^4x \underbrace{\sqrt{-g}}_{g_{\mu\nu}} \left[-\frac{1}{2}\left(M^2 + \xi h^2\right) \underbrace{g^{\mu\nu}}_{g_{\mu\nu}} \underbrace{R_{\mu\nu}}_{\Gamma^{\alpha}_{\beta\gamma}} + \frac{1}{2} \underbrace{g^{\mu\nu}}_{g_{\mu\nu}} \partial_\mu h \partial_\nu h - \frac{\lambda}{4} h^4 \right]$$

Higgs inflation: metric formulation

- Non-minimal coupling to gravity

$$S = \int d^4x \underbrace{\sqrt{-g}}_{g_{\mu\nu}} \left[-\frac{1}{2}\left(M^2 + \xi h^2\right) \underbrace{g^{\mu\nu} R_{\mu\nu}}_{g_{\mu\nu}} + \frac{1}{2} \underbrace{g^{\mu\nu}}_{g_{\mu\nu}} \partial_\mu h \partial_\nu h - \frac{\lambda}{4} h^4 \right]$$

- Weyl transformation to Einstein frame:

$$g_{\mu\nu} = g_{E\mu\nu} \left(1 + \frac{\xi h^2}{M^2}\right)^{-2}$$

$$R_{\mu\nu} = R_{E\mu\nu} + \dots$$

Higgs inflation: metric formulation

- Einstein frame action:

$$S = \int d^4x \sqrt{-g_E} \left[-\frac{1}{2} M^2 g^{E\mu\nu} R_{E\mu\nu} + \right.$$

$$\left. \frac{1}{2} g^{E\mu\nu} \frac{1+\xi h^2/M^2 + 6\xi^2 h^2/M^2}{(1+\xi h^2/M^2)^2} \partial_\mu h \partial_\nu h - \frac{\lambda h^4}{4\left(1+\frac{\xi h^2}{M^2}\right)^2} \right]$$

$$\frac{d\chi}{dh} = \frac{\sqrt{1+\xi h^2 + 6\xi^2 h^2}}{1+\xi h^2}$$

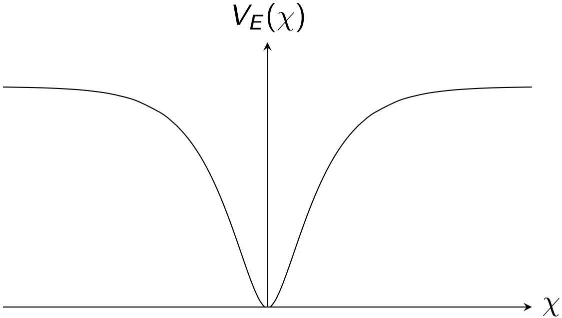
$$V_E(h)$$

Higgs inflation: metric formulation

- Einstein frame action:

$$S = \int d^4x \sqrt{-g_E} \left[-\frac{1}{2} M^2 g^{E\mu\nu} R_{E\mu\nu} + \frac{1}{2} g^{E\mu\nu} \partial_\mu \chi \partial_\nu \chi - V_E(\chi) \right]$$

- Einstein frame potential:



Higgs inflation: Palatini formulation

- Non-minimal coupling to gravity

$$S = \int d^4x \underbrace{\sqrt{-g}}_{g_{\mu\nu}} \left[-\frac{1}{2}\left(M^2 + \xi h^2\right) \underbrace{g^{\mu\nu}}_{g_{\mu\nu}} \underbrace{R_{\mu\nu}}_{\Gamma^{\alpha}_{\beta\gamma}} + \frac{1}{2} \underbrace{g^{\mu\nu}}_{g_{\mu\nu}} \partial_\mu h \partial_\nu h - \frac{\lambda}{4} h^4 \right]$$

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Higgs inflation: Palatini formulation

- Einstein frame action:

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Higgs inflation: Palatini formulation

- Einstein frame action:

$$S = \int d^4x \sqrt{-g_E} \left[-\frac{1}{2} M^2 g^{E\mu\nu} R_{E\mu\nu} + \frac{1}{2} g^{E\mu\nu} \partial_\mu \chi \partial_\nu \chi - V_E(\chi) \right]$$

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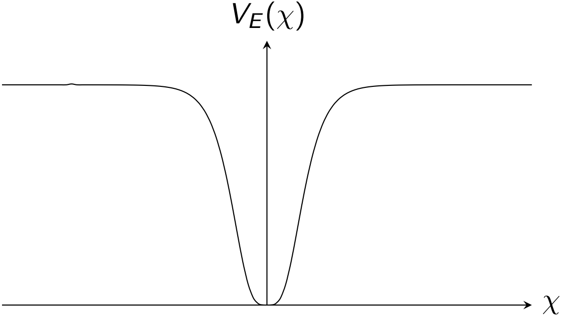


Metric vs. Palatini

- Metric: [0710.3755]

$$\xi \approx 800\sqrt{\lambda}N \sim 10^4 \quad (N = 50, \lambda \sim 0.1)$$

$$n_s \approx 1 - \frac{2}{N} \approx 0.96\,, \quad r \approx \frac{12}{N^2} \approx 4.8 \times 10^{-3}$$

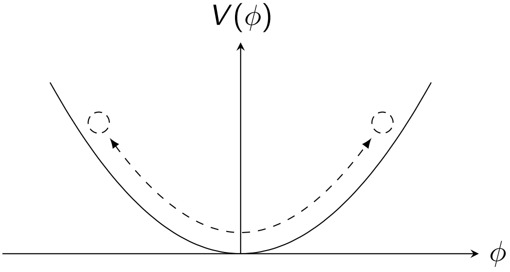
- Palatini: [0803.2664]

$$\xi \approx 3.8 \times 10^6 \lambda N^2 \sim 10^9$$

$$n_s \approx 1 - \frac{2}{N} \approx 0.96\,, \quad r \approx \frac{2}{\xi N^2} \sim 10^{-12}$$

Reheating: Overview

- After inflation, inflaton oscillates around its minimum



$$\ddot{\chi} + 3H\dot{\chi} + V'(\chi) = 0$$

friction oscillation

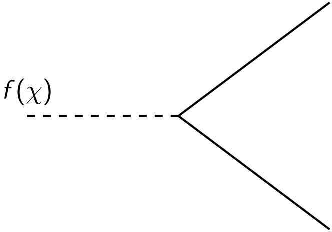
Reheating: Particle production

- Need to transfer some of the energy density in the inflaton condensate to particles

- This happens through interactions of the inflaton field

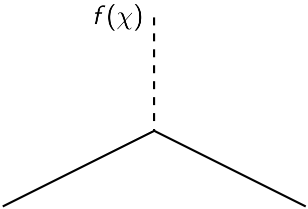
Higgs reheating

- Interactions known
- Decays of Higgs condensate (subdominant)



Higgs reheating

- Higgs-induced mass terms for fermions, weak gauge bosons and Higgs bosons



$$m_W^2 = \frac{g^2 h^2}{4(1+\xi h^2/M^2)}\,, \qquad m_Z^2 = \frac{(g^2+g'^2)h^2}{4(1+\xi h^2/M^2)}\,,$$

$$m_t^2 = \frac{y_t^2 h^2}{2(1+\xi h^2/M^2)}\,, \qquad m_h^2 = V_E''(h)$$

Time-dependent mass: Preheating

- Scalar field mode function:

$$\ddot{Q}_k + 3H\dot{Q}_k + \underbrace{\left[\frac{k^2}{a^2} + m^2(\chi)\right]}_{\omega_k^2} Q_k = 0$$

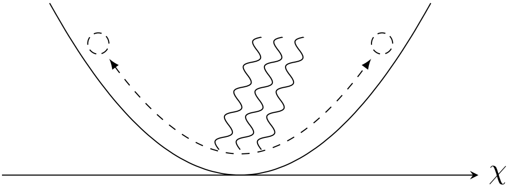
- For constant m, standard solution:

$$a^{3/2} Q_k = \frac{\alpha_k}{\sqrt{2\omega_k}} e^{-i\int^t \omega_k(t')\,dt'} + \frac{\beta_k}{\sqrt{2\omega_k}} e^{i\int^t \omega_k(t')\,dt'}$$

- Adiabatic vacuum: $\alpha_k = 1$, $\beta_k = 0$
- Non-vacuum states: $n_k = |\beta_k|^2$

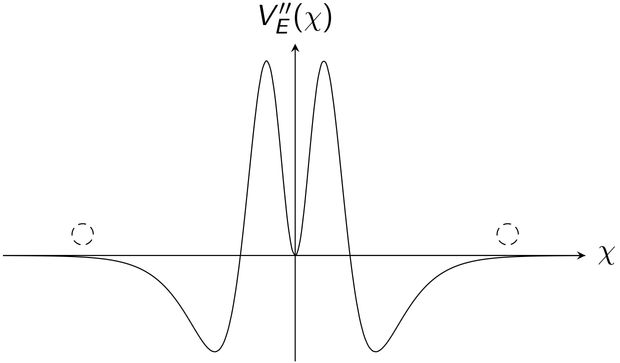
Parametric resonance

- Adiabaticity condition $|\dot{\omega}_k|/\omega_k^2 \gg 1$ broken near the bottom of the Higgs potential
- For certain Fourier modes: explosive particle production



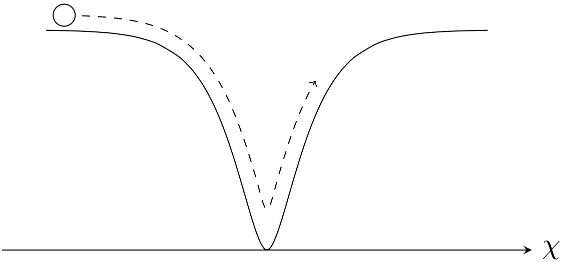
Tachyonic preheating

- If $m^2 < 0 \Rightarrow \omega_k^2 < 0$, mode function grows exponentially: $Q_k \propto e^{\sqrt{-\omega_k^2}t}$
- Possible for Higgs perturbations with $m_h^2 = V_E''$



Higgs preheating: Metric case

$$\ddot{\chi} + 3H\dot{\chi} + V'(\chi) = 0$$



- Quick decay of χ oscillation amplitude; then, $V_E = M\chi^2$, M=constant

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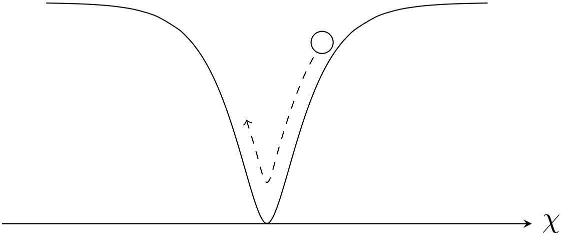

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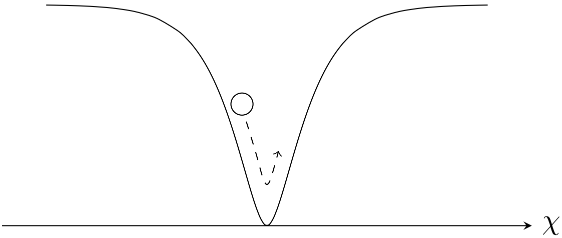


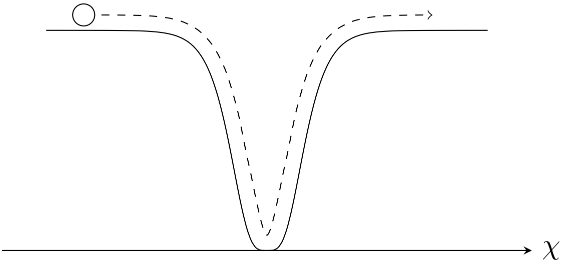
- Quick decay of χ oscillation amplitude; then, $V_E = M\chi^2$, M=constant

Higgs preheating: Metric case

- Higgs mass squared constant and positive: no production
- Parametric resonance: production of W, Z bosons
  - At early times, decay to fermions (*Combined reheating*)
  - Later, bosons start to accumulate
- Reheating takes a few e-folds

Higgs preheating: Palatini case

$$\ddot{\chi} + 3H\dot{\chi} + V'(\chi) = 0$$



- Oscillation amplitude stays almost constant! Why?

Higgs preheating: Palatini case

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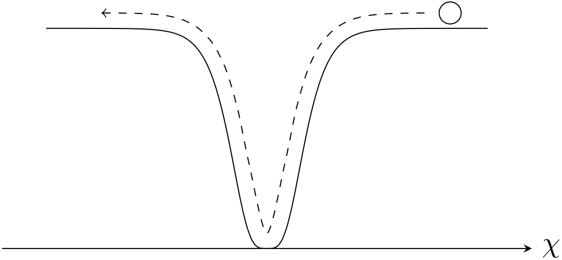

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Higgs preheating: Palatini case

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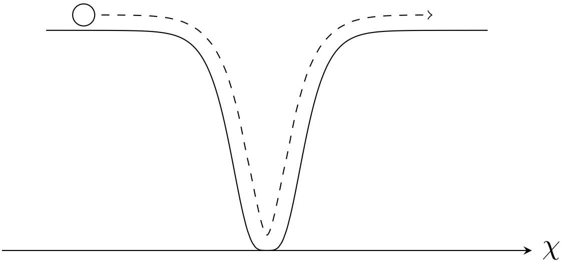

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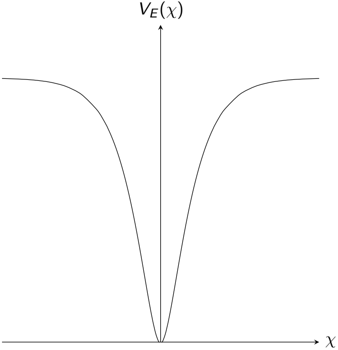
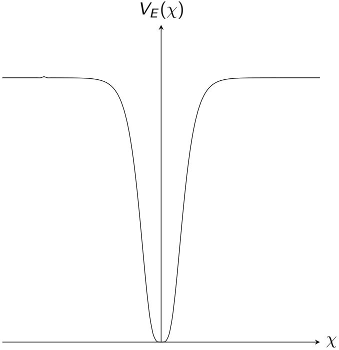
Comparison: Metric vs Palatini



$$\frac{d\chi}{dh} = \frac{\sqrt{1+\xi h^2 + 6\xi^2 h^2}}{1+\xi h^2}$$



$$\frac{d\chi}{dh} = \frac{\sqrt{1+\xi h^2}}{1+\xi h^2}$$

Comparison: Metric vs Palatini

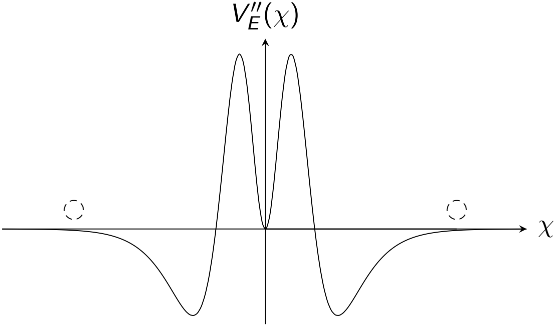
- Oscillation energy depleted more quickly in metric formulation:

$$M_P^2 \frac{dH}{dh} = -\frac{\dot{\chi}^2}{2\dot{h}} = -\frac{1}{2}\sqrt{6H^2M_P^2 - 2V_E[\chi(h)]}\,\frac{d\chi}{dh}$$

Bigger in metric case

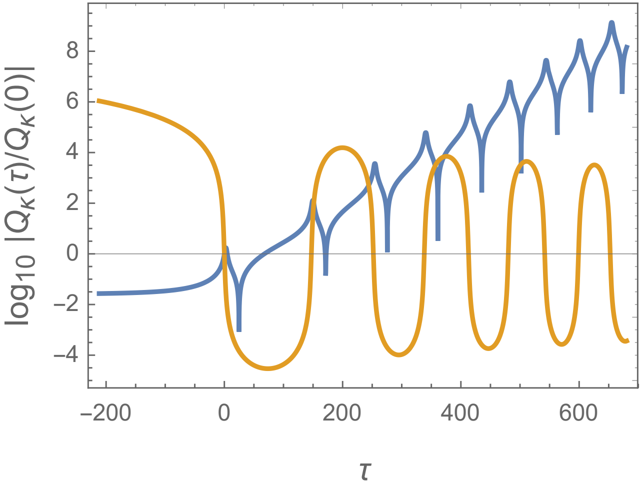
Higgs preheating: Palatini case

- Result: m_h^2 time-dependent, negative at times
- Tachyonic Higgs production is the leading preheating channel



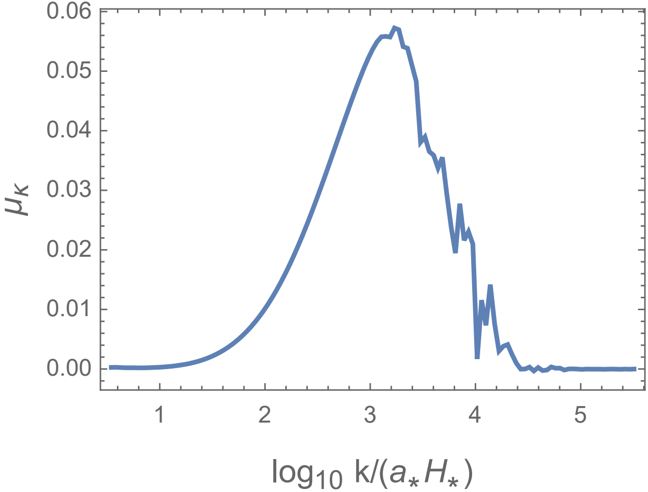
Higgs preheating: Palatini case

- Typical tachyonic mode function: $Q_k \propto e^{\mu_k \tau}$



Higgs preheating: Palatini case

- Growth index depends on k:



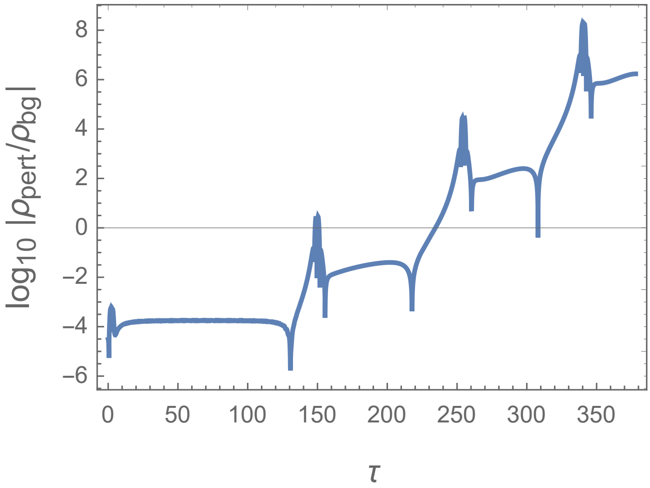
Higgs preheating: Palatini case

- Particle concept is ill-defined, but we can calculate energy density in perturbations:

$$\frac{\rho_{\rm pert}}{\rho_{\rm B}} = \frac{1}{3H^2 M_P^2} \int_{k_{min}}^{k_{max}} \frac{dk\, 4\pi k^2}{(2\pi)^3} \frac{1}{2} \left[|\dot{Q}_k|^2 + \left(\frac{k^2}{a^2} + V_E''(\chi) \right) |Q_k|^2 \right]$$

- Numerical calculations: significant fraction of energy density in perturbations after only a few oscillations, less than one e-fold

Higgs preheating: Palatini case



Significance on cosmology

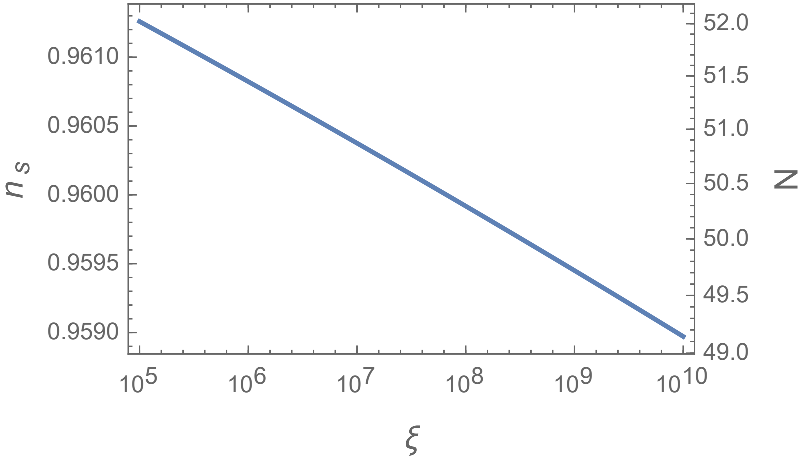
- Palatini formulation: almost instant reheating
- Reheating temperature $T_{\rm RH} = \left(\frac{30\,\lambda}{4\pi^2\xi^2 g_{*\,\rm RH}}\right)^{1/4} M_P$
- E-folds of inflation from CMB pivot scale
  $N_* = 54.9 - \frac{1}{4}\log\xi$

Significance on cosmology

- Spectral index n_s: add 2nd order SR correction and a correction related to SR approximation of N to get

$$n_s \approx 1 - \frac{2}{N_*} - \frac{0.8}{N_*^2}$$

Significance on cosmology



Caveats

- Non-linear behaviour?
- Details of backreaction?
- Thermalization?
- Quantum corrections to potential ignored

Summary

- Preheating in Palatini Higgs inflation is very efficient due to tachyonic Higgs production
- Detailed knowledge of reheating is needed to make accurate cosmological predictions

$$\epsilon_V \approx \frac{1}{8\xi N_V^2}\,, \quad \eta_V \approx -\frac{1}{N_V}\,, \quad \xi_V \approx \frac{1}{N_V^2}$$

$$n_s \approx 1 - 6\epsilon_V + 2\eta_V + \frac{1}{3}(44-18c)\epsilon_V^2 + (4c-14)\epsilon_V\eta_V + \frac{2}{3}\eta_V^2 + \frac{1}{6}(13-3c)\zeta_V$$

$$N_V \equiv \int_i^\chi \frac{d\chi}{\sqrt{2\epsilon_V}}$$

$$N_* \approx N_V + 1.8$$

$$n_s \approx 1 - \frac{2}{N_V} + \frac{2.8}{N_V^2} \approx 1 - \frac{2}{N_*} - \frac{0.8}{N_*^2}$$

	Peak			Plateau		
ξ	$n_{\rm osc}$	ΔN	$\frac{d \ln \rho_{\rm pert}}{dn_{\rm osc}}$	$n_{\rm osc}$	ΔN	$\frac{d \ln \rho_{\rm pert}}{dn_{\rm osc}}$
10^5	1.75	0.10	10.3	2	0.07	10.1
10^6	1.25	0.04	12.7	1.5	0.03	12.3
10^7	1.25	0.02	15.0	1.5	0.02	14.2
10^8	0.75	0.007	17.1	1.5	0.009	16.0
10^9	0.75	0.004	19.3	1	0.003	17.9

ξ	$\frac{g_2^2}{\lambda}$	$\left\langle \frac{\Delta\rho_F}{\rho_{\rm B}} \right\rangle$
10^5	27789	0.064
10^6	2814	0.037
10^7	285	0.022
10^8	29	0.014
10^9	2.9	0.014