



# Eternal inflation in primordial black hole models

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COSMOLOGY FROM HOME 2025

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# Primordial black holes

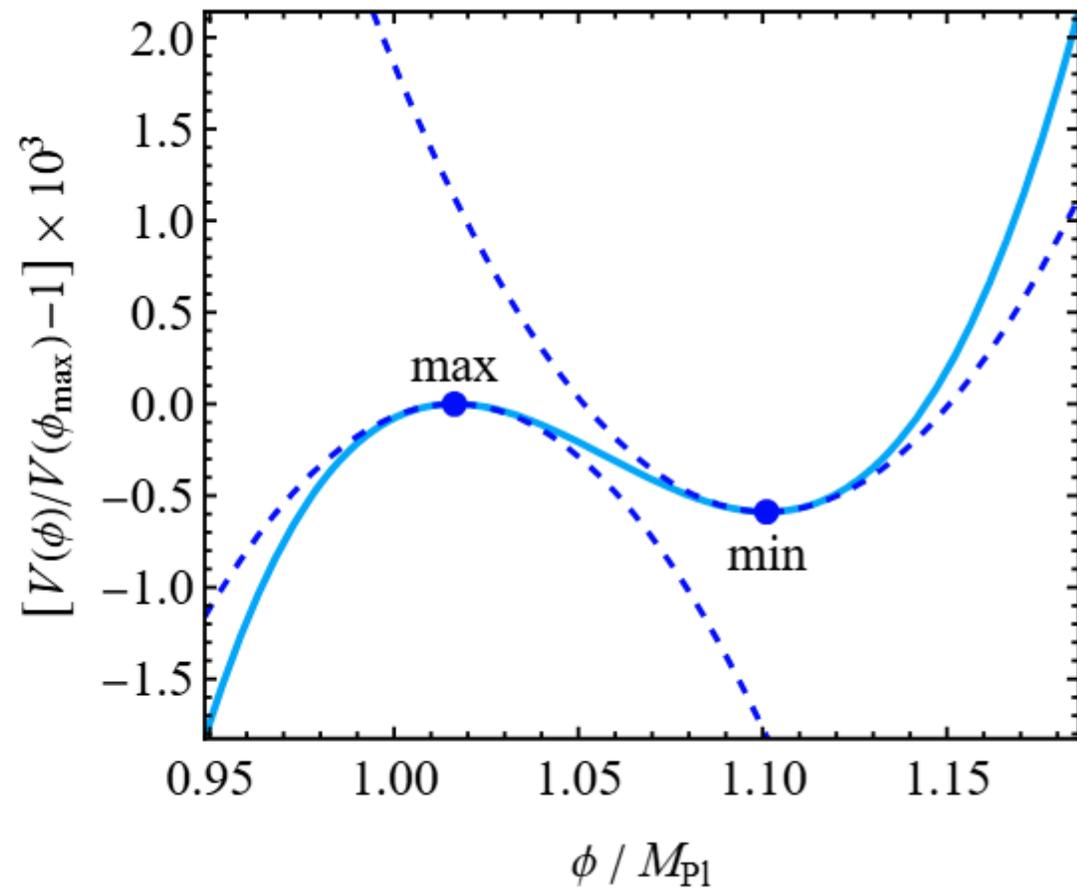
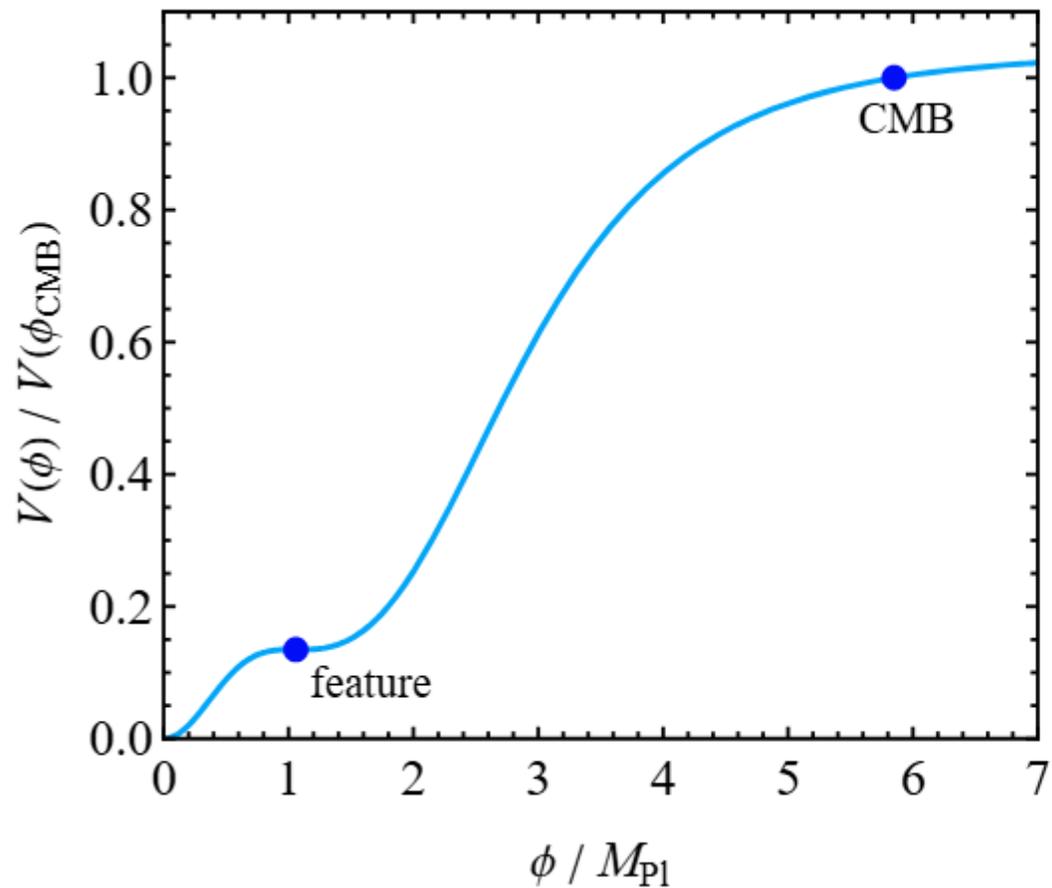
Black holes from primordial processes

Dark matter candidate

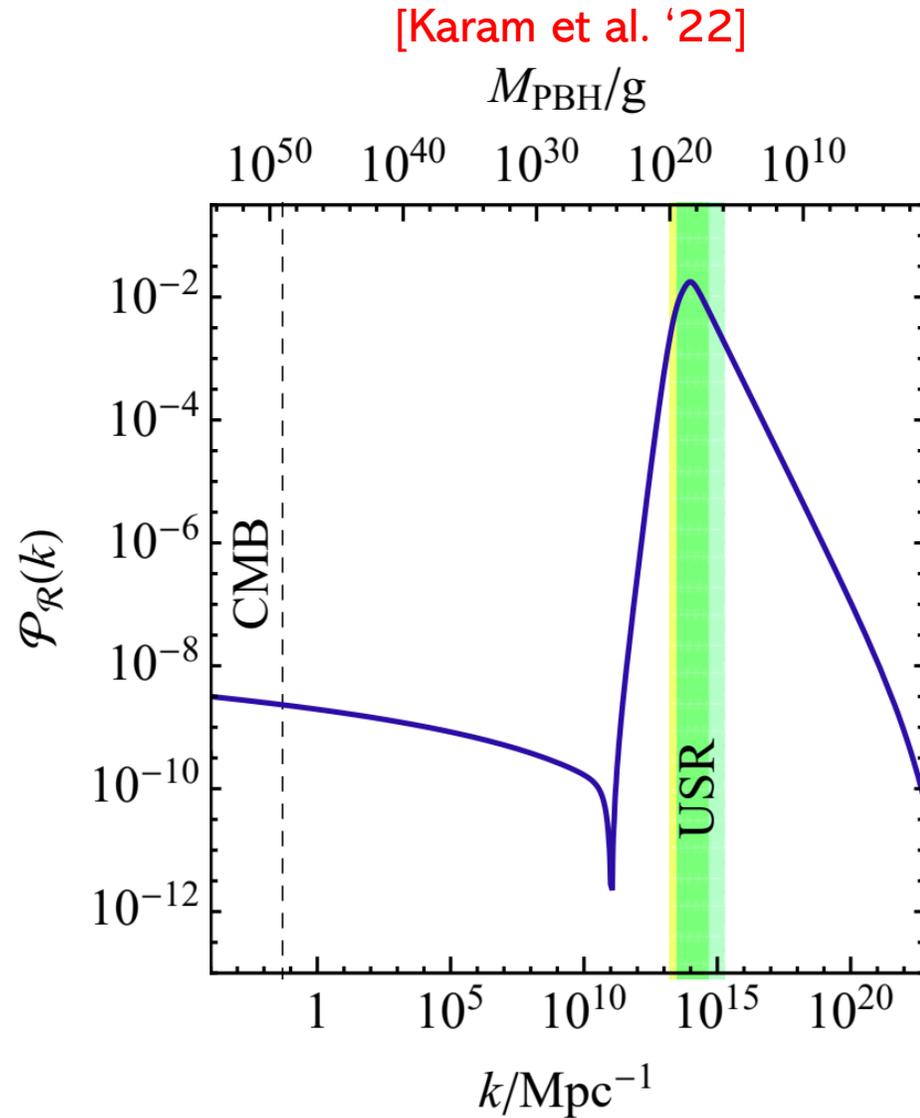
Gravitational wave source

# Inflection point inflation

[Tomberg & Dimopoulos '25], under preparation



# Inflection point inflation



# Eternal inflation

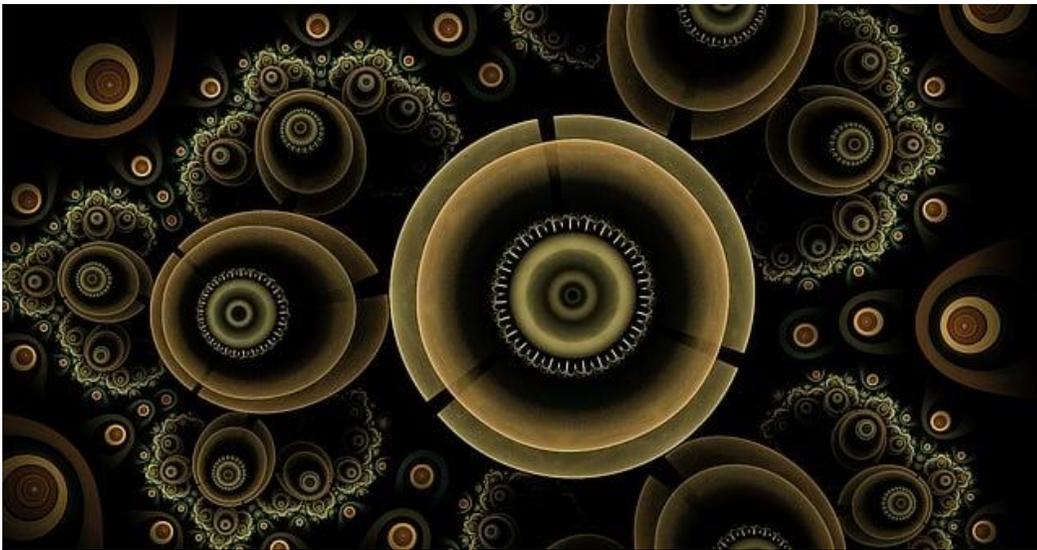
Large quantum fluctuations  
counteract classical drift

Inflating regions grow fast

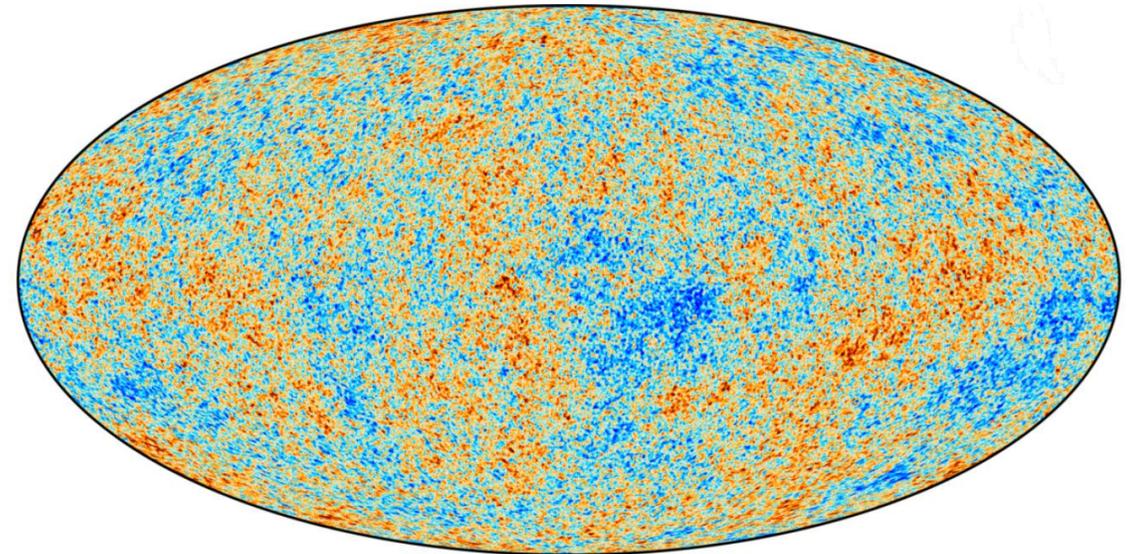
Inflation is eternal: 'most' of  
the Universe always inflating

# Why interesting?

Global structure?



Observational predictions?



# Eternal inflation quantified

Inflating volume non-zero at late times

$$\lim_{N \rightarrow \infty} \int_{\phi_{\text{end}}}^{\infty} e^{3N} P(\phi, N) d\phi > 0$$

# Solving $P(\phi, N)$

Fokker-Planck equation:

$$\partial_N P(\phi, N) = \partial_\phi \left[ \partial_\phi \left( \frac{1}{2} \sigma^2(\phi) P(\phi, N) \right) - \mu(\phi) P(\phi, N) \right]$$

with absorbing boundary condition  $P(\phi_{\text{end}}, N) = 0$   
at end-of-inflation hypersurface

# $P(\phi, N)$ asymptotics

$$P(\phi, N) \sim e^{-\lambda N}$$

Exponential tails!  
E.g. [Ezquiaga et al. '18]

$$\int_{\phi_{\text{end}}}^{\infty} e^{3N} P(\phi, N) d\phi \sim e^{(3-\lambda)N}$$

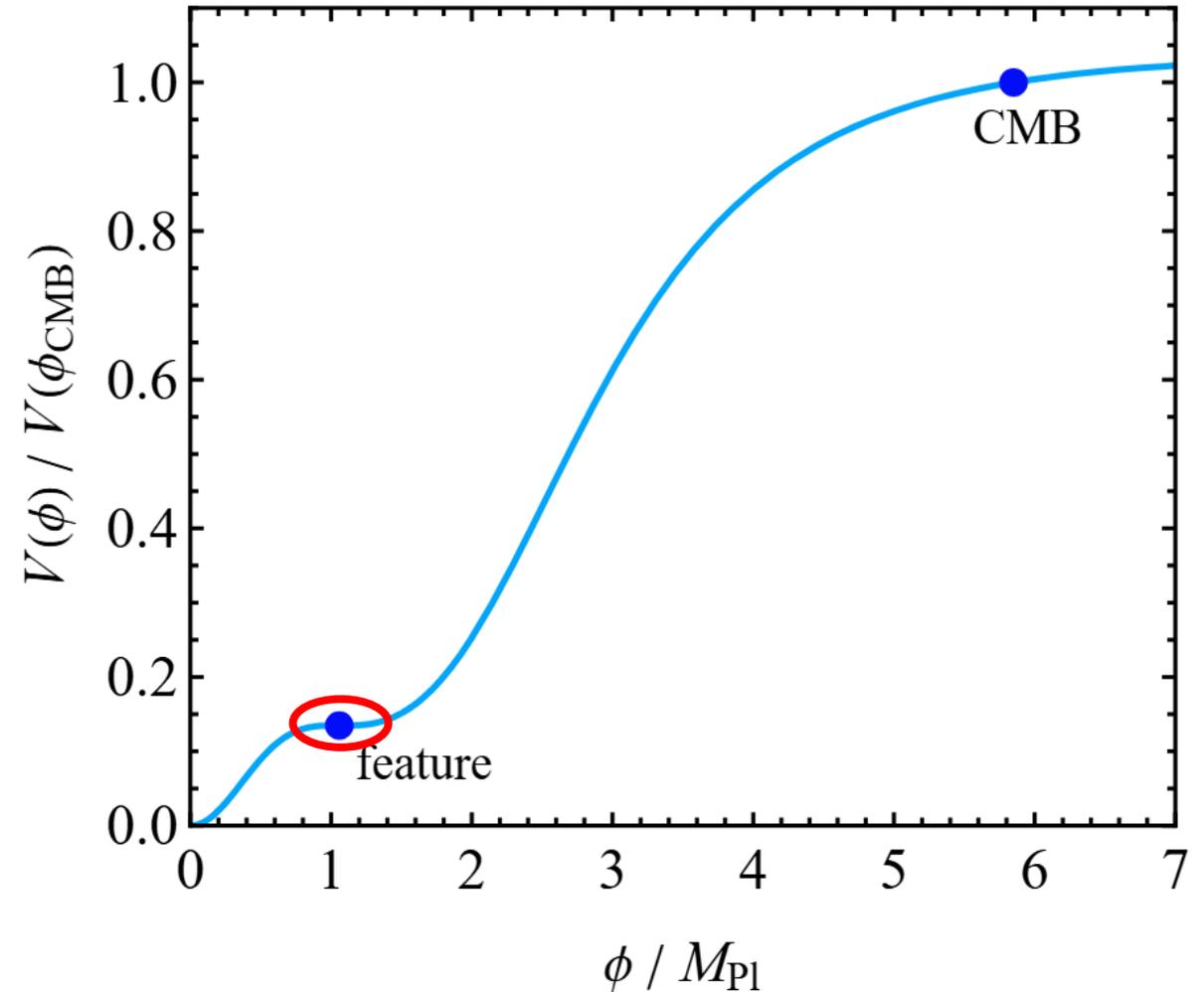
Eternal inflation  $\iff \lambda \leq 3$

# Practical problems

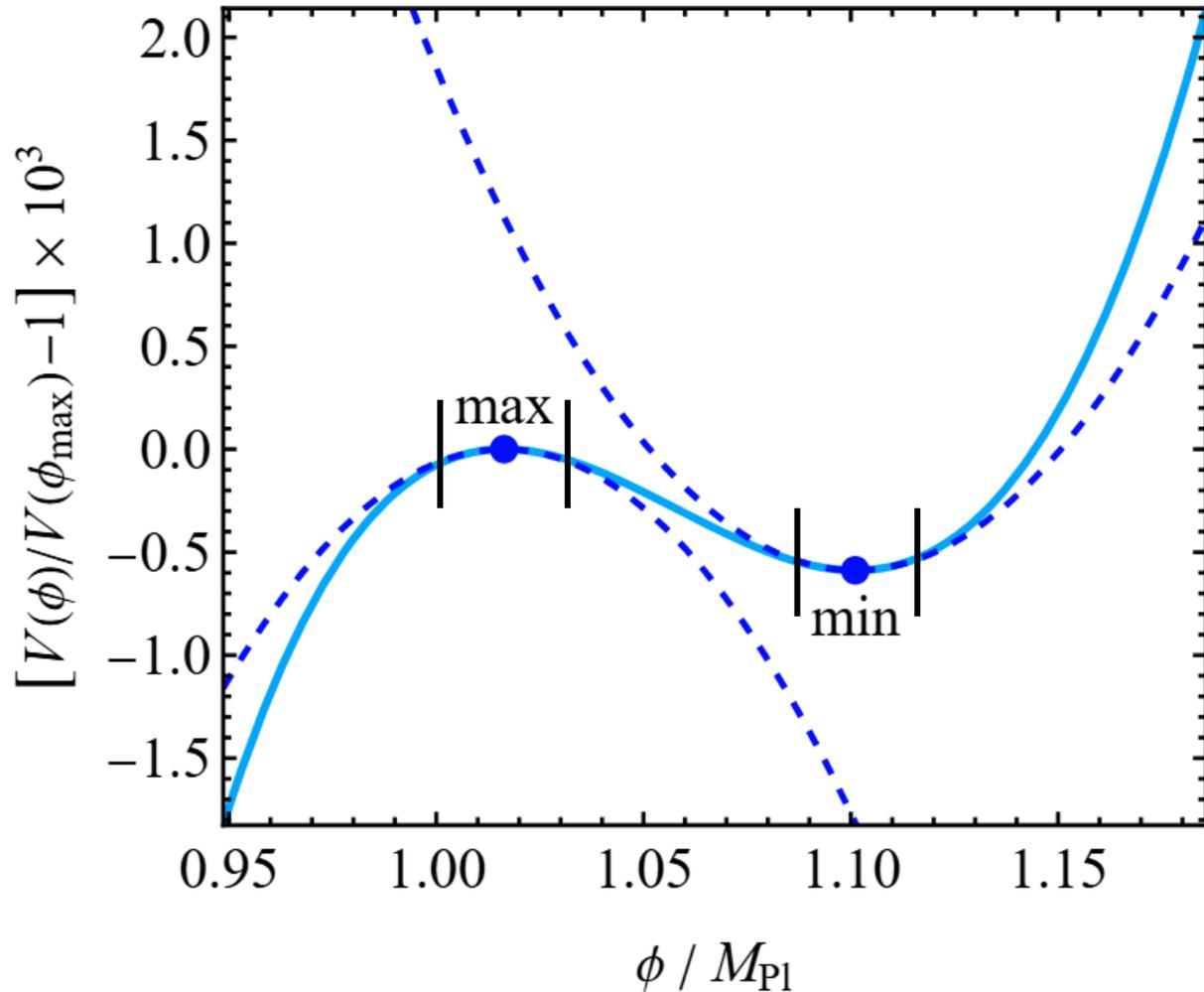
Features in potential  
on a short field interval

Usually eternal inflation  
at large field values

Focus on the feature



# Zoom into the feature



Parabolic approximation:

$$\underline{\sigma} = \frac{H}{2\pi} \times \frac{\Gamma(\frac{3}{2} - \eta_H)}{\Gamma(\frac{3}{2})} \times \left(\frac{2}{\sigma_c}\right)^{-\eta_H}$$

$$\underline{\mu} = -\eta_H(\phi - \phi_i)$$

Absorbing boundaries

$$\text{at } \phi = \phi_i \pm \underline{\phi}_b$$

# Wide limit

$$\begin{array}{ccc} & \phi_b \gtrsim \sigma & \\ \nearrow & & \nwarrow \\ \sim M_{\text{Pl}} & & \sim H \end{array}$$

Maximum:

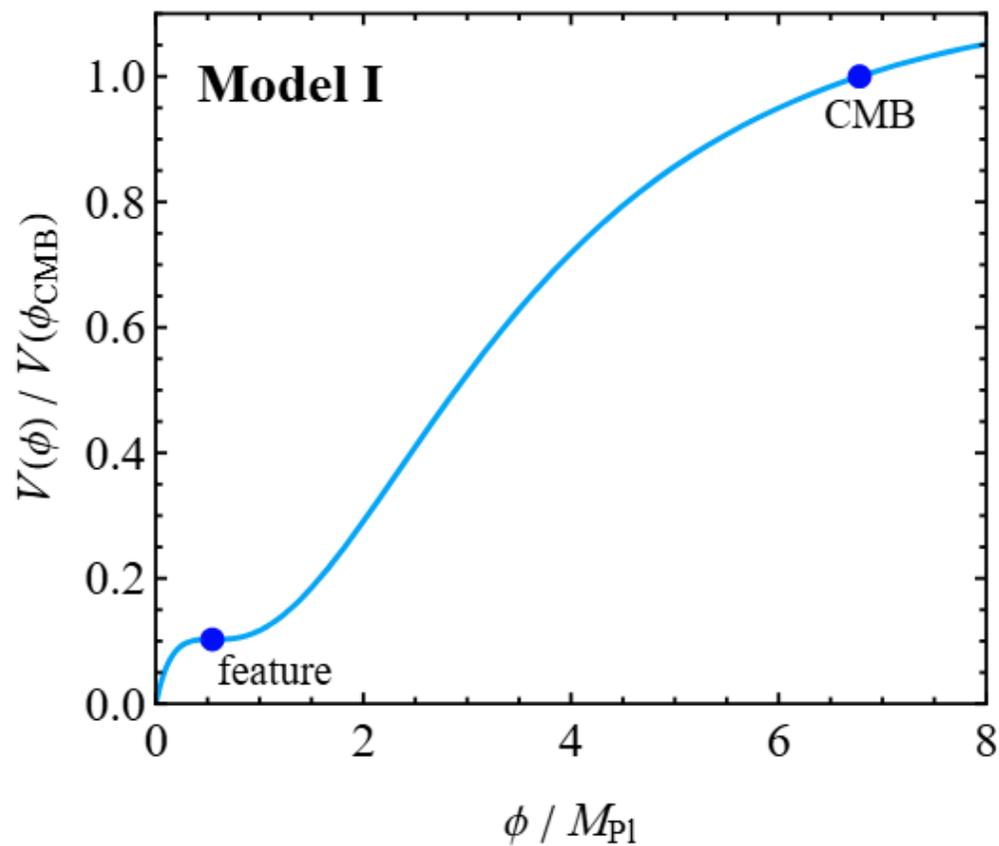
$$\lambda \approx |\eta_H| \sim 0.1$$

Minimum:

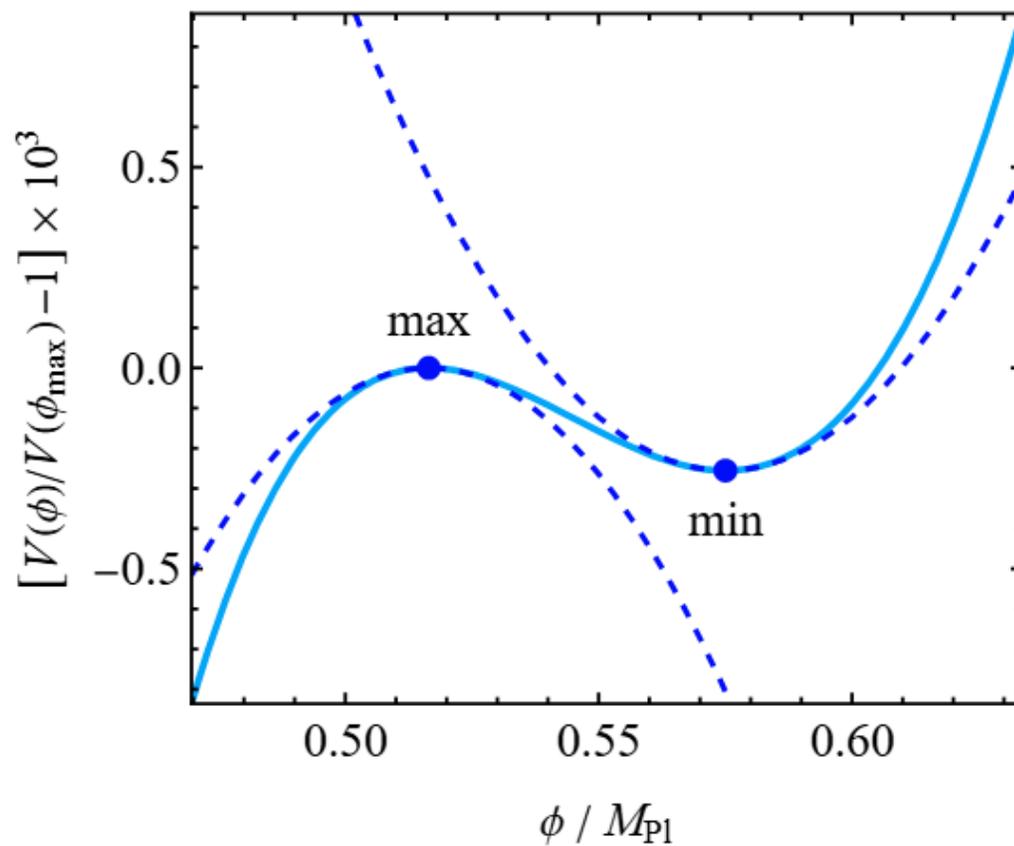
$$\lambda \approx 0$$

# Typical potentials:

[Kannike et al. '17]



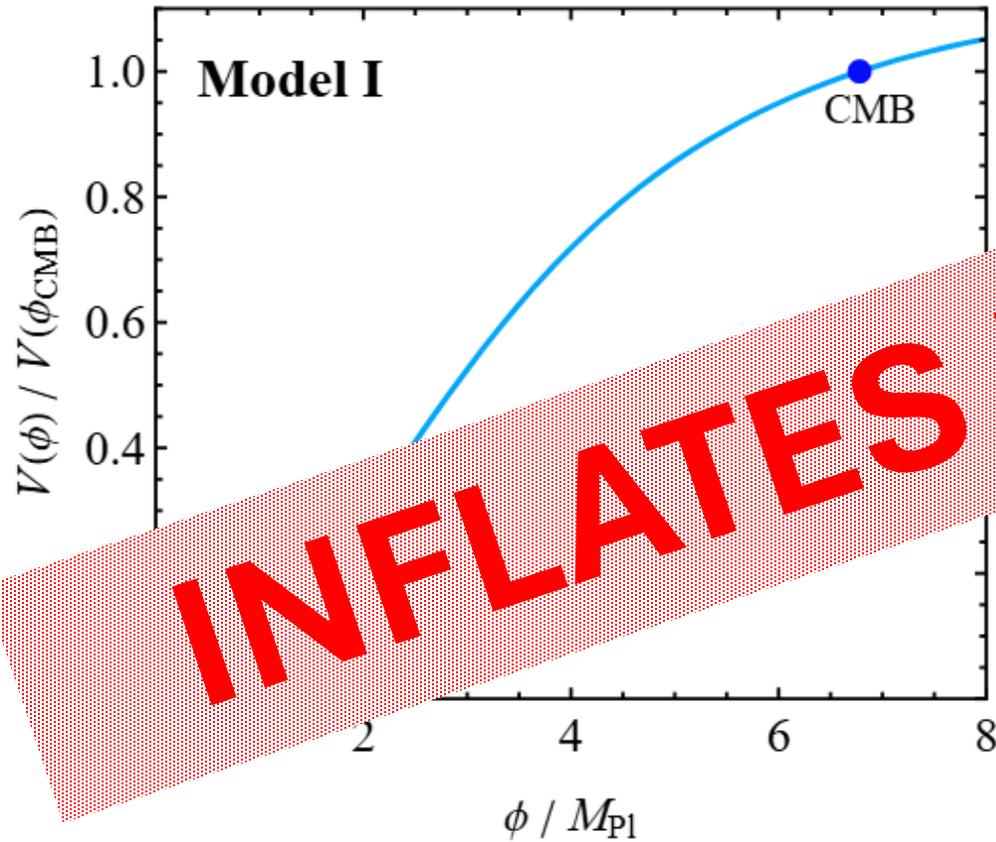
max:  $\phi_b^2 / \sigma^2 \sim 10^4$ ,  $\lambda = -\eta_H = 0.413$



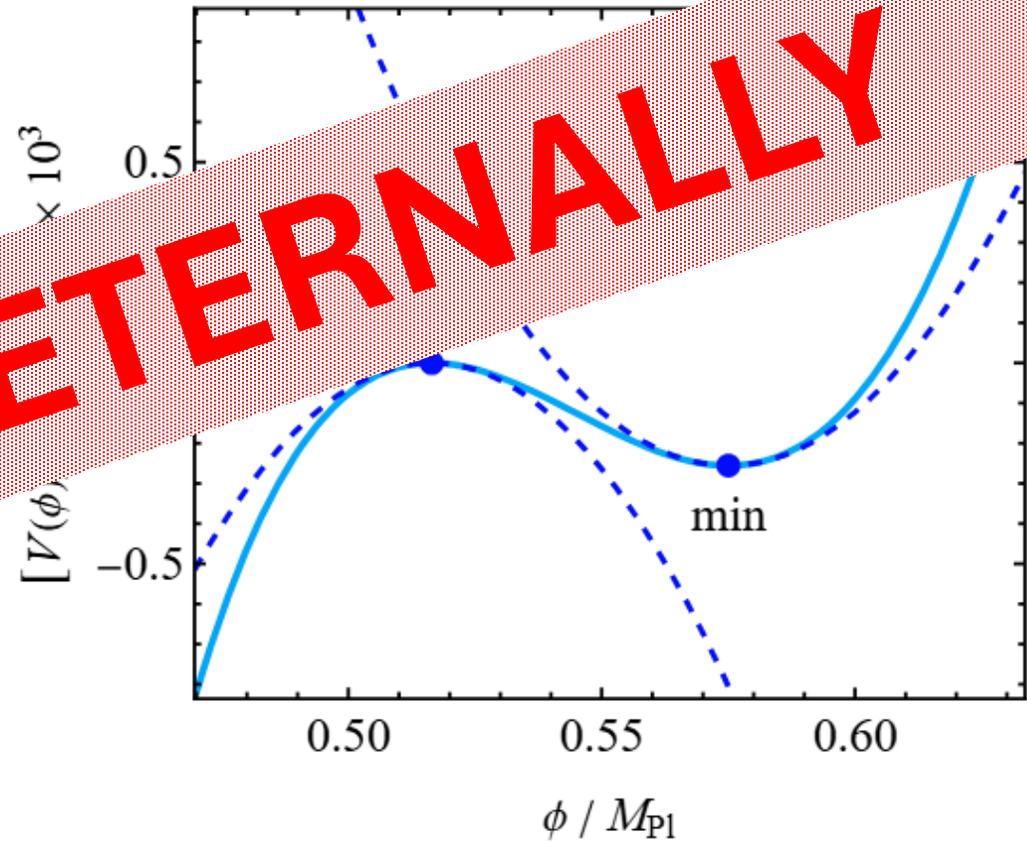
min:  $\phi_b^2 / \sigma^2 \sim 10^8$ ,  $\lambda \approx 0$

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[Kannike et al. '17]



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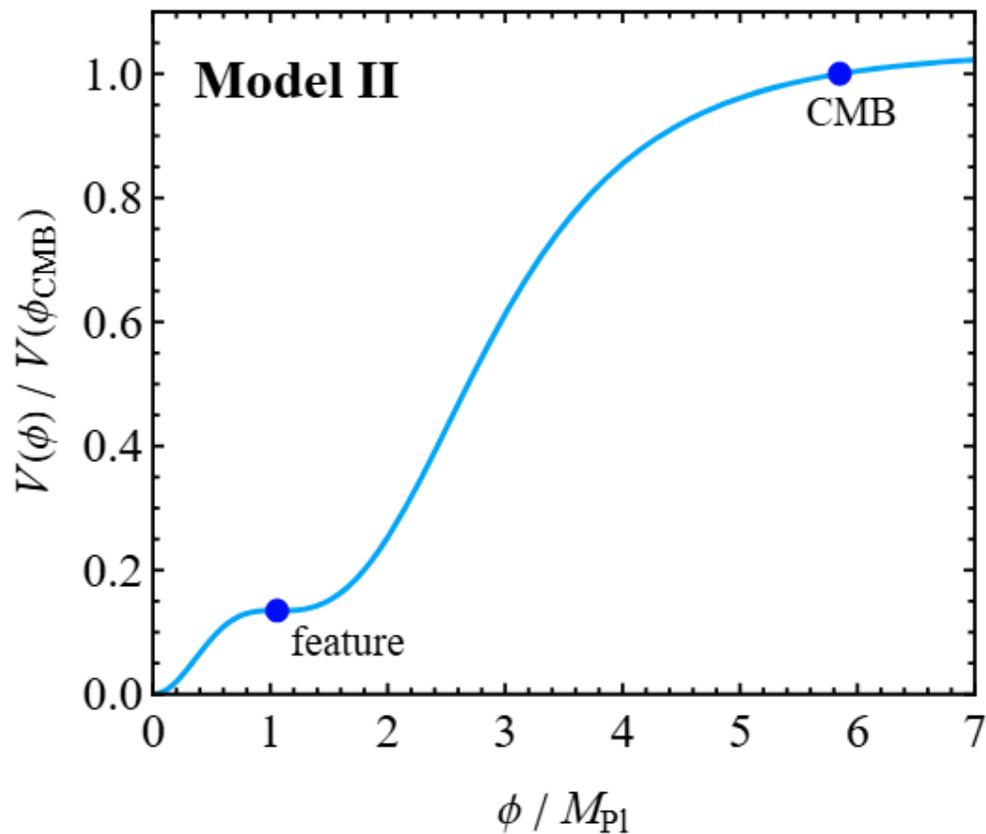


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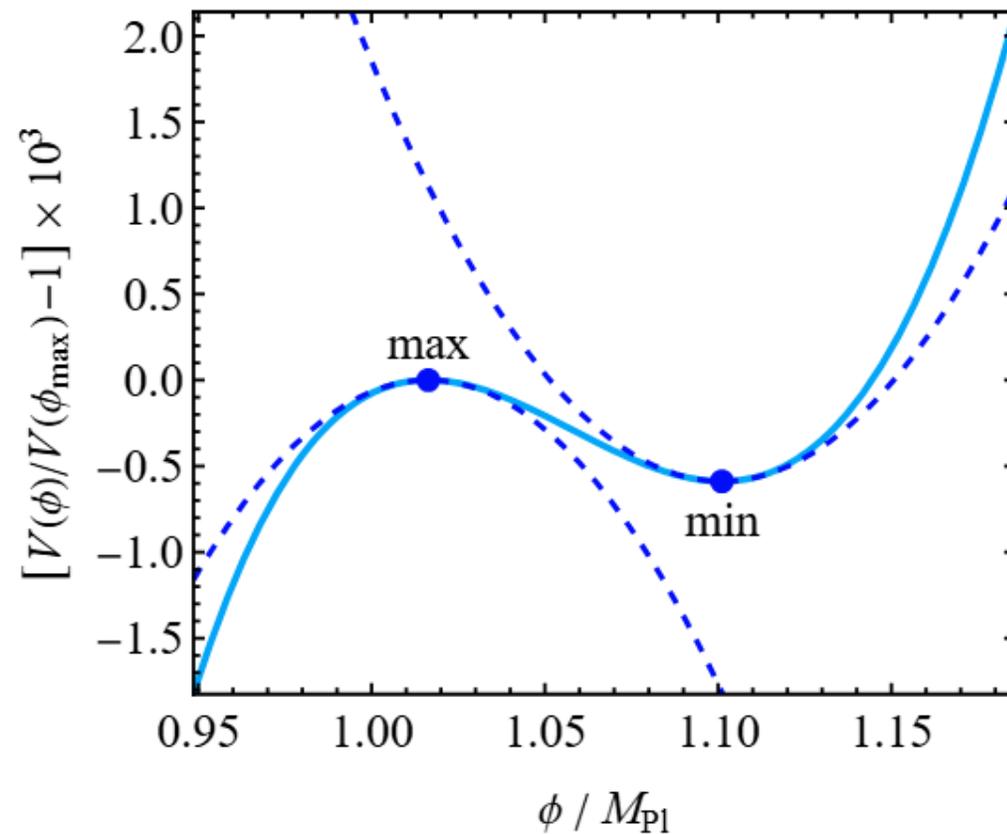
**INFLATES ETERNALLY**

# Typical potentials:

[Dalianis et al. '18]



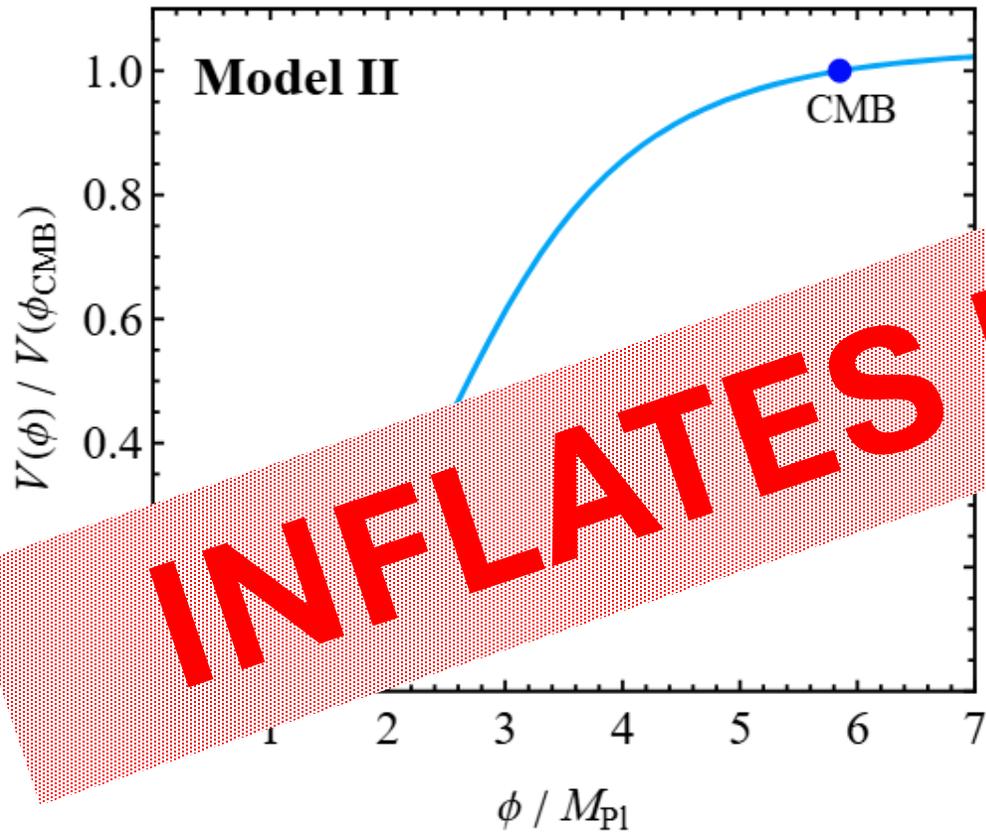
max:  $\phi_b^2 / \sigma^2 \sim 10^4$ ,  $\lambda = -\eta_H = 0.441$



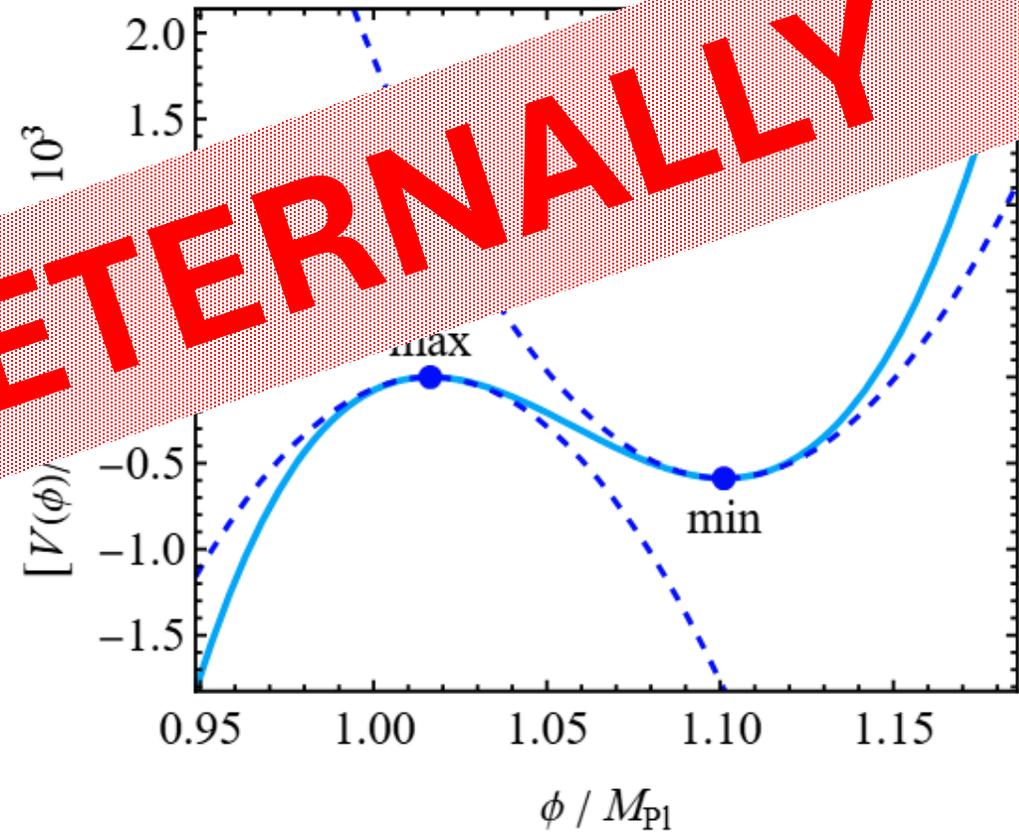
min:  $\phi_b^2 / \sigma^2 \sim 10^9$ ,  $\lambda \approx 0$

# Typical potentials:

[Dalianis et al. '18]



max:  $\phi_b^2 / \sigma^2 \sim 10^4$ ,  $\lambda = -\eta_H = \underline{\underline{0.441}}$

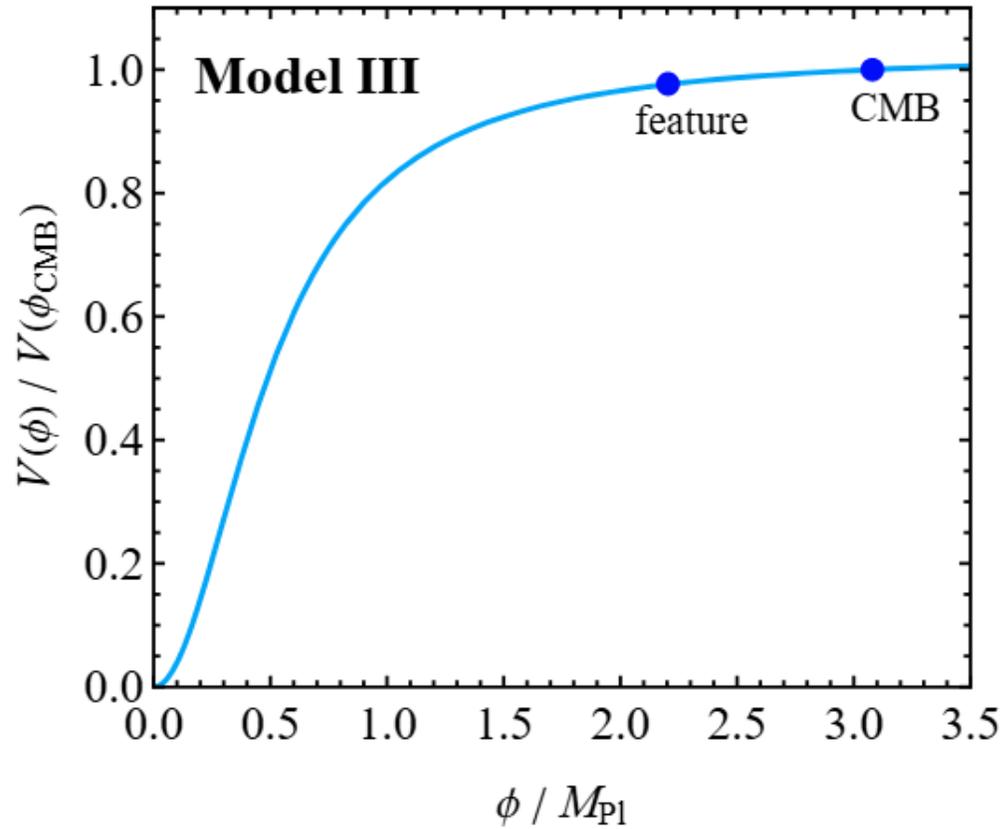


min:  $\phi_b^2 / \sigma^2 \sim 10^9$ ,  $\lambda \approx \underline{\underline{0}}$

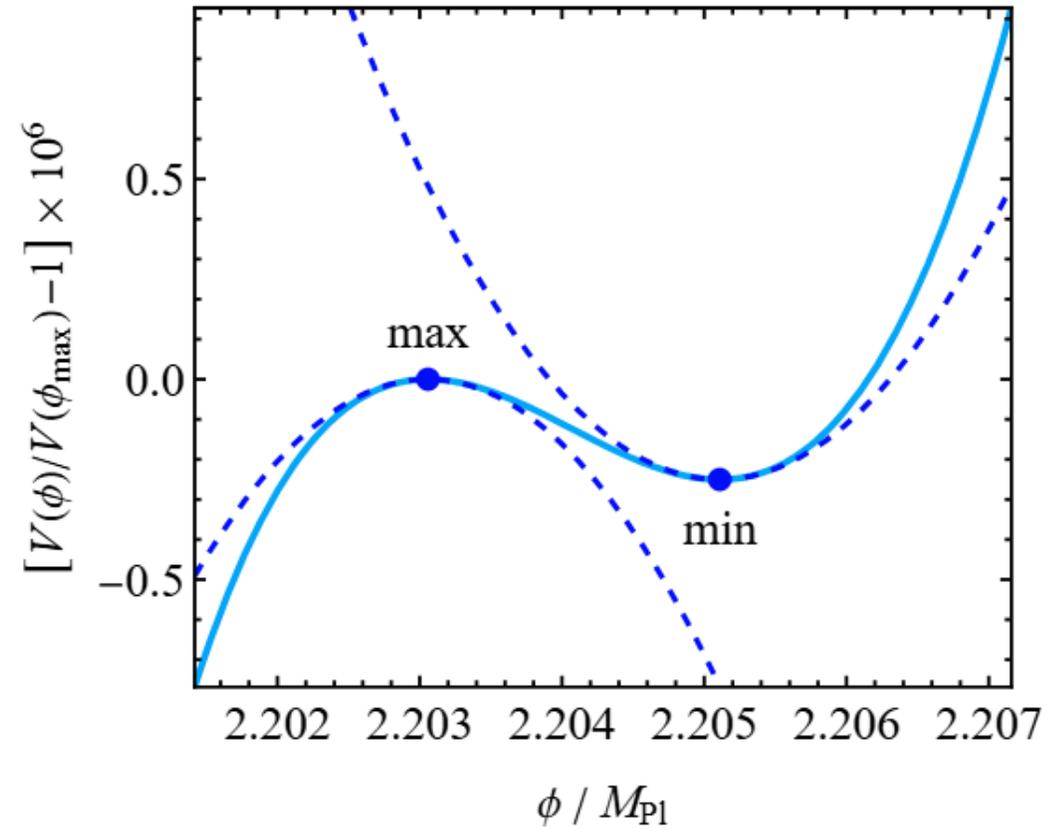
**INFLATES ETERNALLY**

# Typical potentials:

[Mishra & Sahni '19]



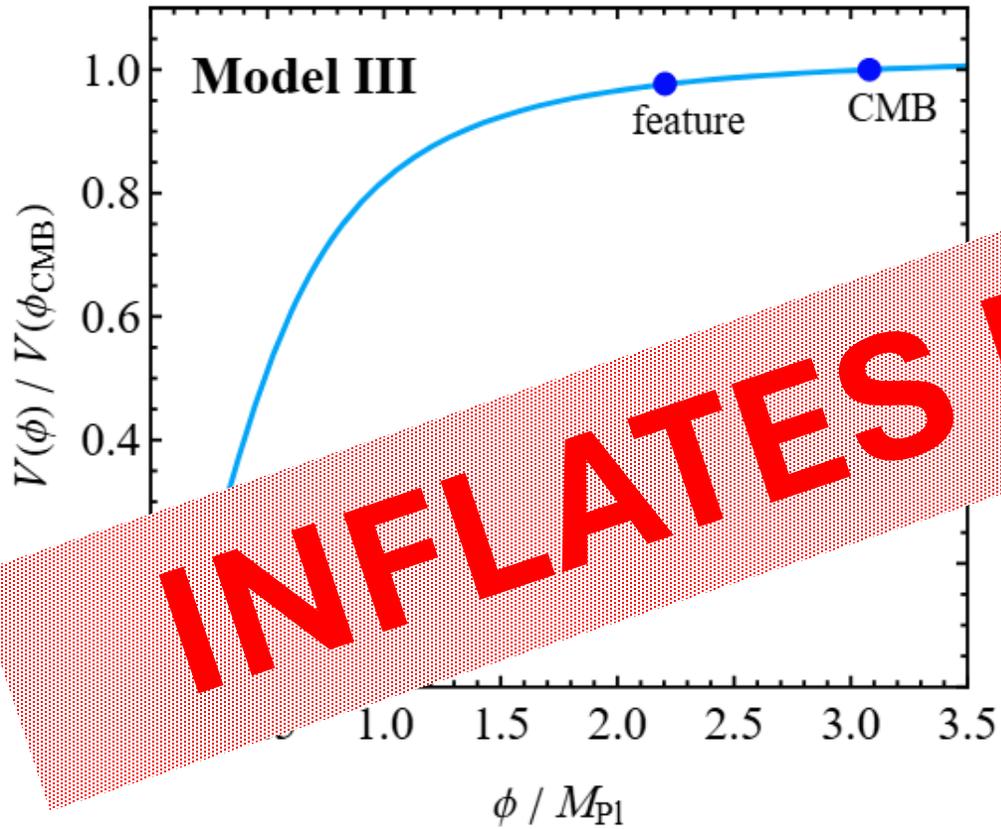
max:  $\phi_b^2 / \sigma^2 \approx 40$ ,  $\lambda = -\eta_H = 0.329$



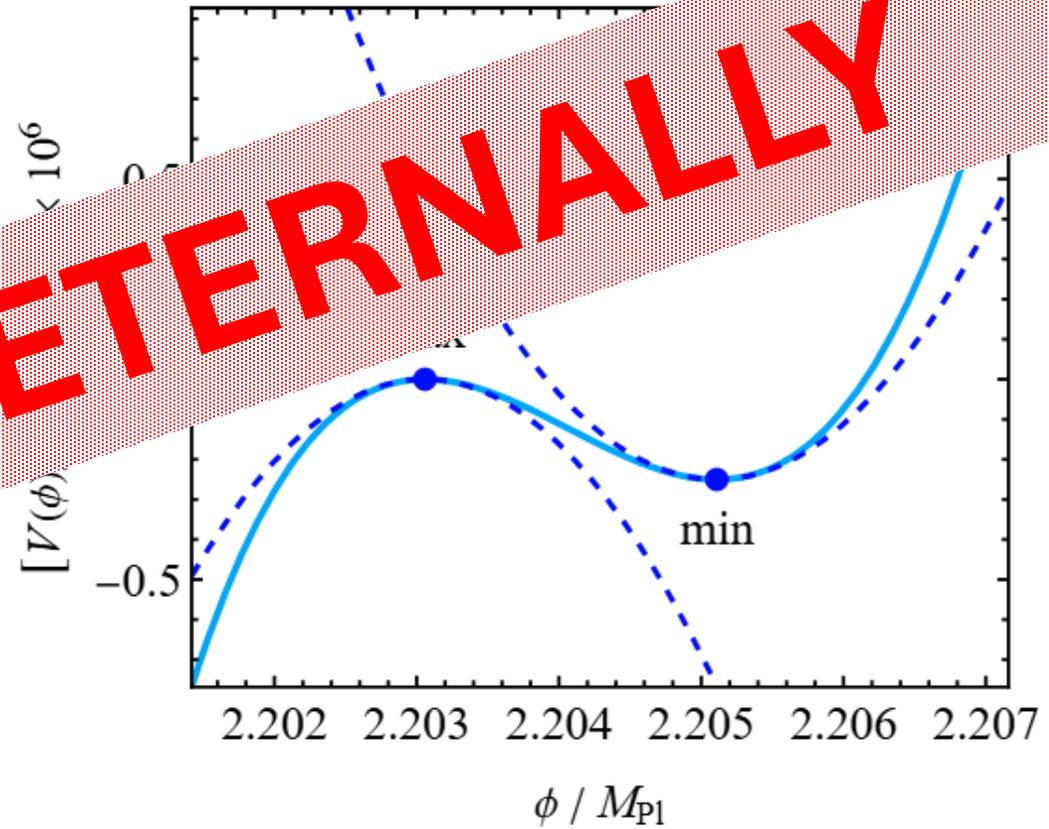
min:  $\phi_b^2 / \sigma^2 \sim 10^5$ ,  $\lambda \approx 0$

# Typical potentials:

[Mishra & Sahni '19]



max:  $\phi_b^2 / \sigma^2 \approx 40$ ,  $\lambda = -\eta_H = \underline{\underline{0.329}}$



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**INFLATES ETERNALLY**

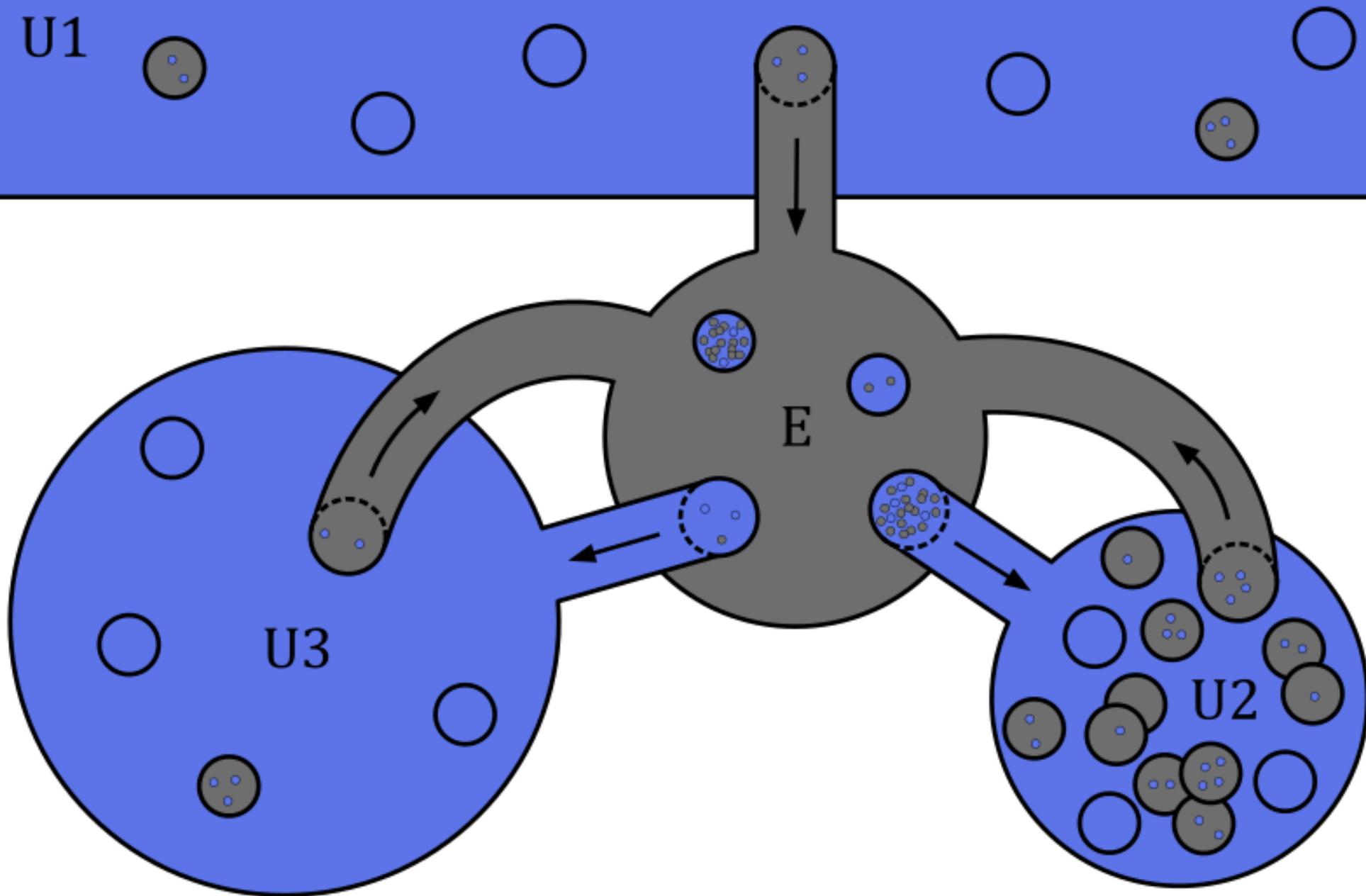
Eternal inflation is generic!

# Structure of space-time

Eternal inflation:  
inside “extreme” black holes (type II)

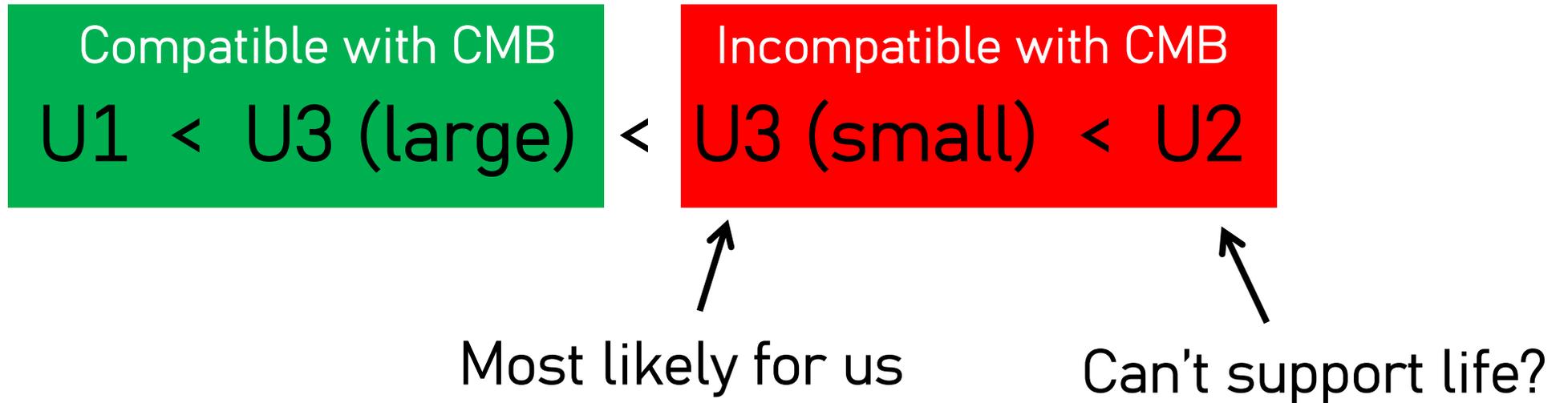
What does the Universe look like  
inside these black holes?

U1



# Which universe do we live in?

Volume weighted probabilities:



Eternally inflating PBH models  
incompatible with the CMB! \*

# \* If volume weighting is used

U1 and U3 (large) still exist,  
only with small volume fraction

If volume weighting abandoned: eternal  
inflation can't solve initial conditions for  
inflation, either

Eternal inflation is generic in  
inflection point PBH models

Models incompatible with CMB?

Volume weighting?

# Parabolic approximation

Kummer's equation; lowest eigenvalue from

$${}_1F_1\left(-\frac{\lambda_1}{2\eta_H}; \frac{1}{2}; \frac{\phi_b^2}{\sigma^2}\eta_H\right) = 0$$

Wide limit:  $\phi_b^2 \gtrsim \sigma^2 \implies \lambda_1 \approx \begin{cases} |\eta_H|, & \eta_H < 0 \\ 0, & \eta_H > 0 \end{cases}$

$$\eta_H = \frac{3}{2} \left( 1 - \sqrt{1 - \frac{4}{3} \eta_V} \right) .$$