Eternal inflation in primordial black hole models

COBALT, INSTITUT PASCAL, 3 JULY 2025

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Primordial black holes

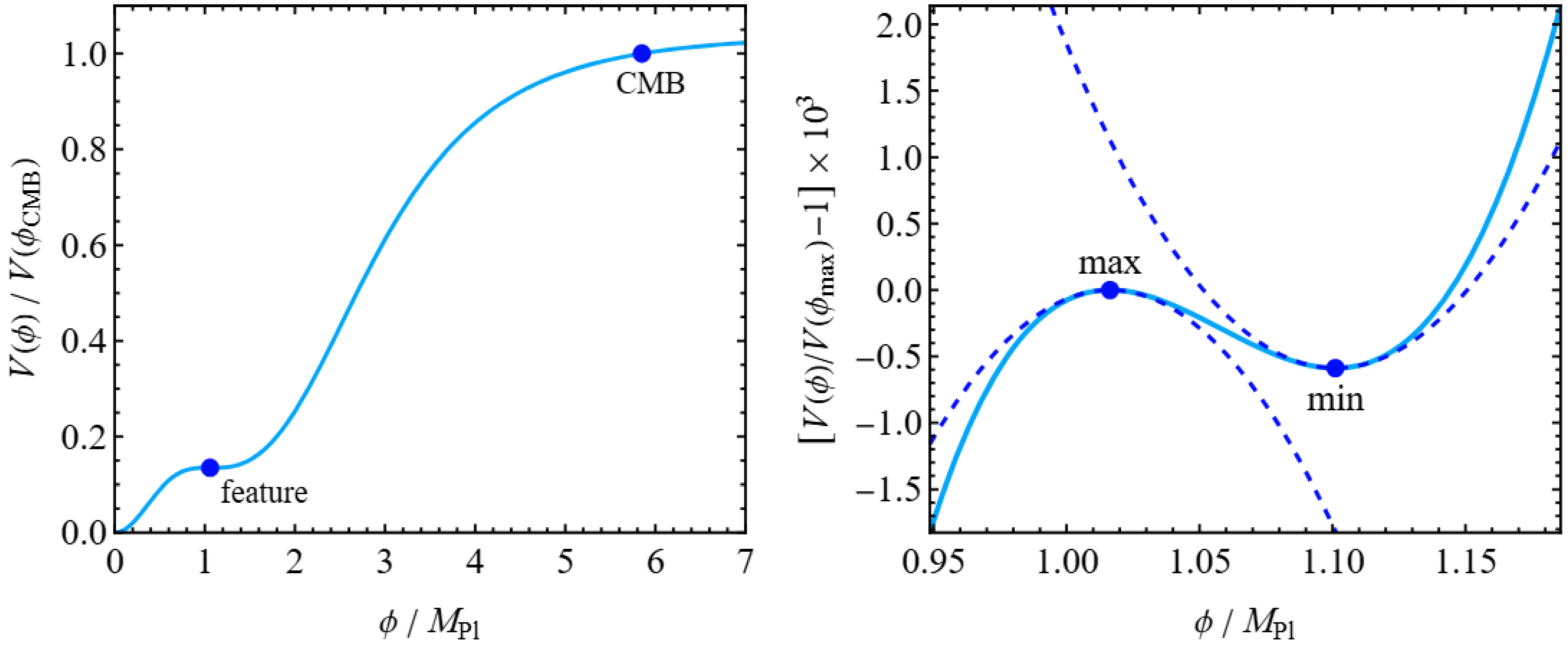
Black holes from primordial processes

Dark matter candidate

Gravitational wave source

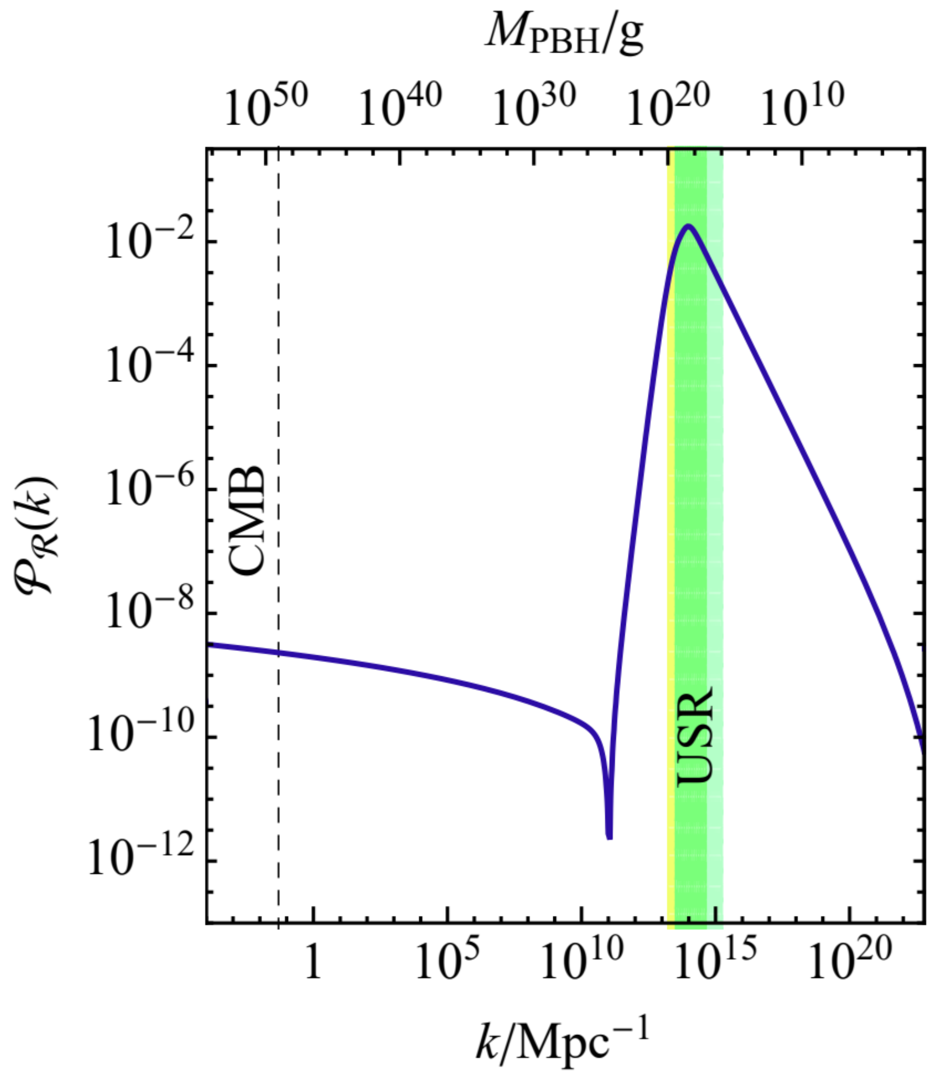
Inflection point inflation

[Tomberg & Dimopoulos '25], under preparation



Inflection point inflation

[Karam et al. '22]



Eternal inflation

Large quantum fluctuations
counteract classical drift

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Inflating regions grow fast

Eternal inflation

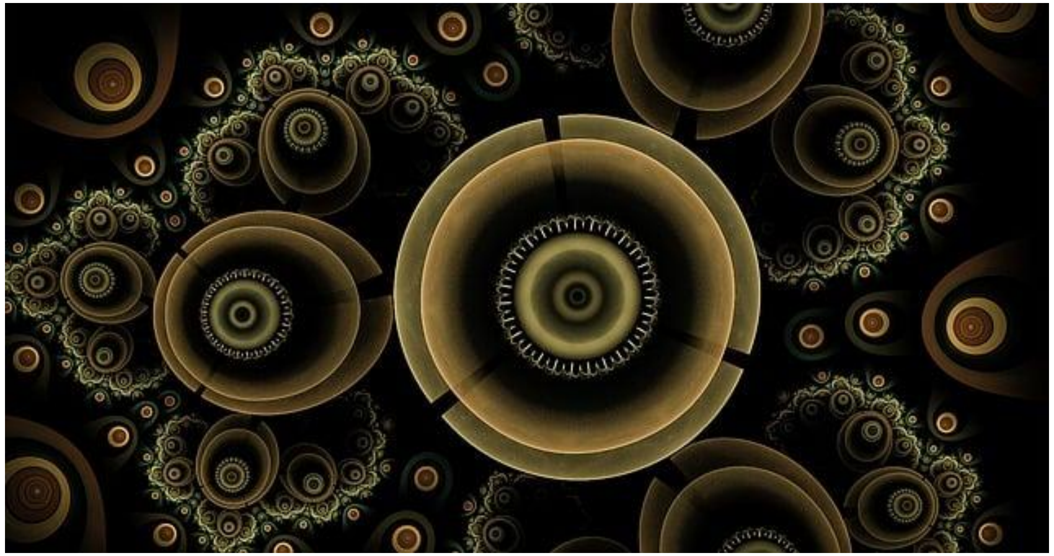
Large quantum fluctuations counteract classical drift

Inflating regions grow fast

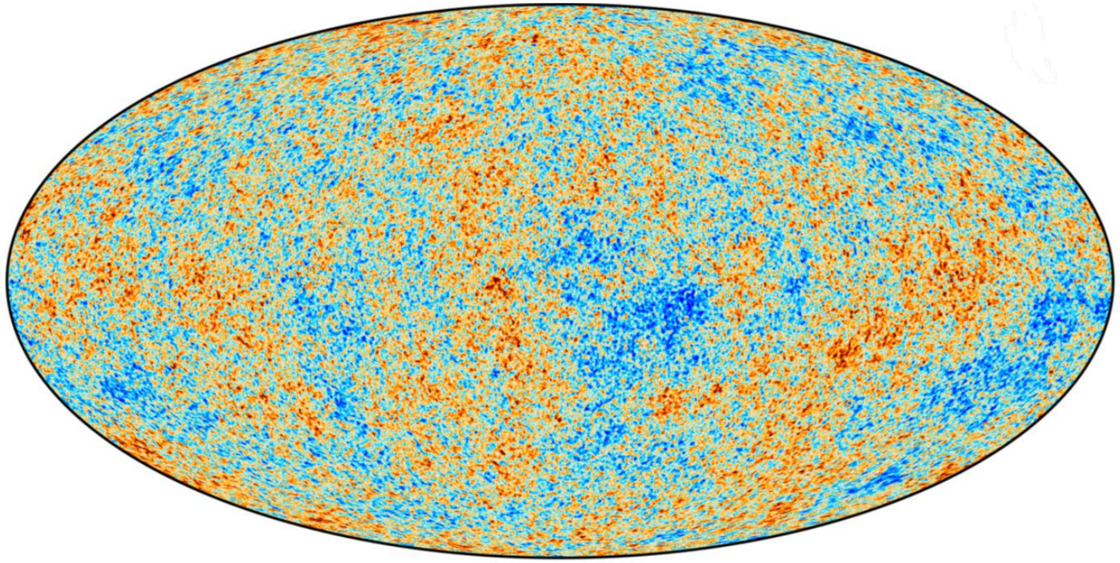
Inflation is eternal: ‘most’ of the Universe always inflating

Why interesting?

Global structure?



Observational predictions?



Eternal inflation quantified

Inflating volume non-zero at late times

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$$P(\phi, N)$$

Eternal inflation quantified

Inflating volume non-zero at late times

$$e^{3N}P(\phi, N)$$

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$$\int_{\phi_{\mathrm{end}}}^{\infty} e^{3N} P(\phi, N)\,\mathrm{d}\phi$$

Eternal inflation quantified

Inflating volume non-zero at late times

$$\lim_{N\to\infty}\int_{\phi_{\mathrm{end}}}^{\infty} e^{3N}P(\phi,N)\mathrm{d}\phi > 0$$

Solving $P(\phi, N)$

Fokker-Planck equation:

$$\partial_N P(\phi, N) = \partial_\phi \left[\partial_\phi \left(\frac{1}{2} \sigma^2(\phi) P(\phi, N) \right) - \mu(\phi) P(\phi, N) \right]$$

with absorbing boundary condition $P(\phi_{\mathrm{end}}, N) = 0$ at end-of-inflation hypersurface

$P(\phi, N)$ asymptotics

$$P(\phi, N) \sim e^{-\lambda N}$$

Exponential tails!
E.g. [Ezquiaga et al. '18]

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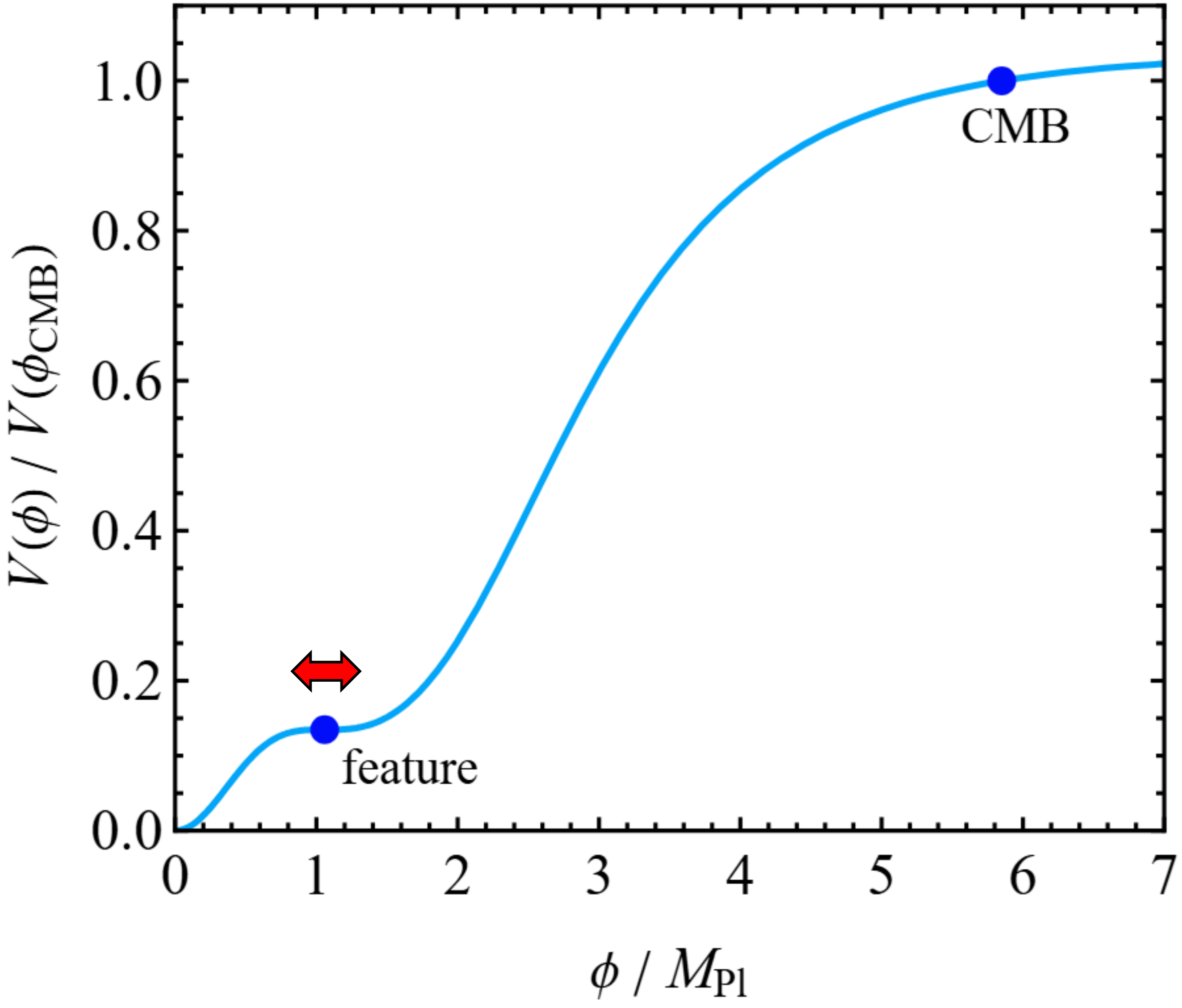
$$\int_{\phi_{\mathrm{end}}}^{\infty} e^{3N} P(\phi, N) \mathrm{d}\phi \sim e^{(3-\lambda)N}$$

Eternal inlation $\iff \lambda \leq 3$

Practical problems

Features in potential
on a short field interval



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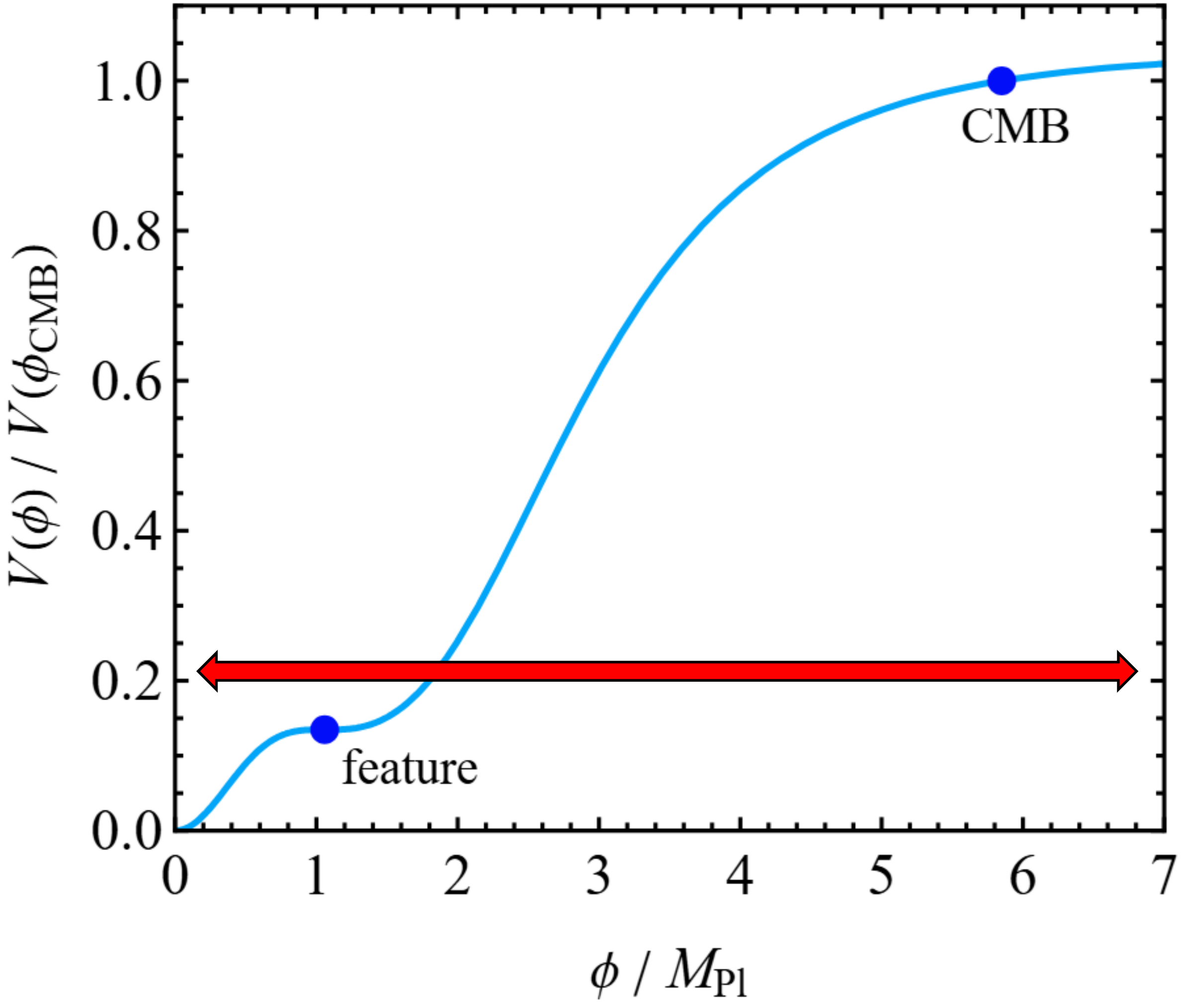
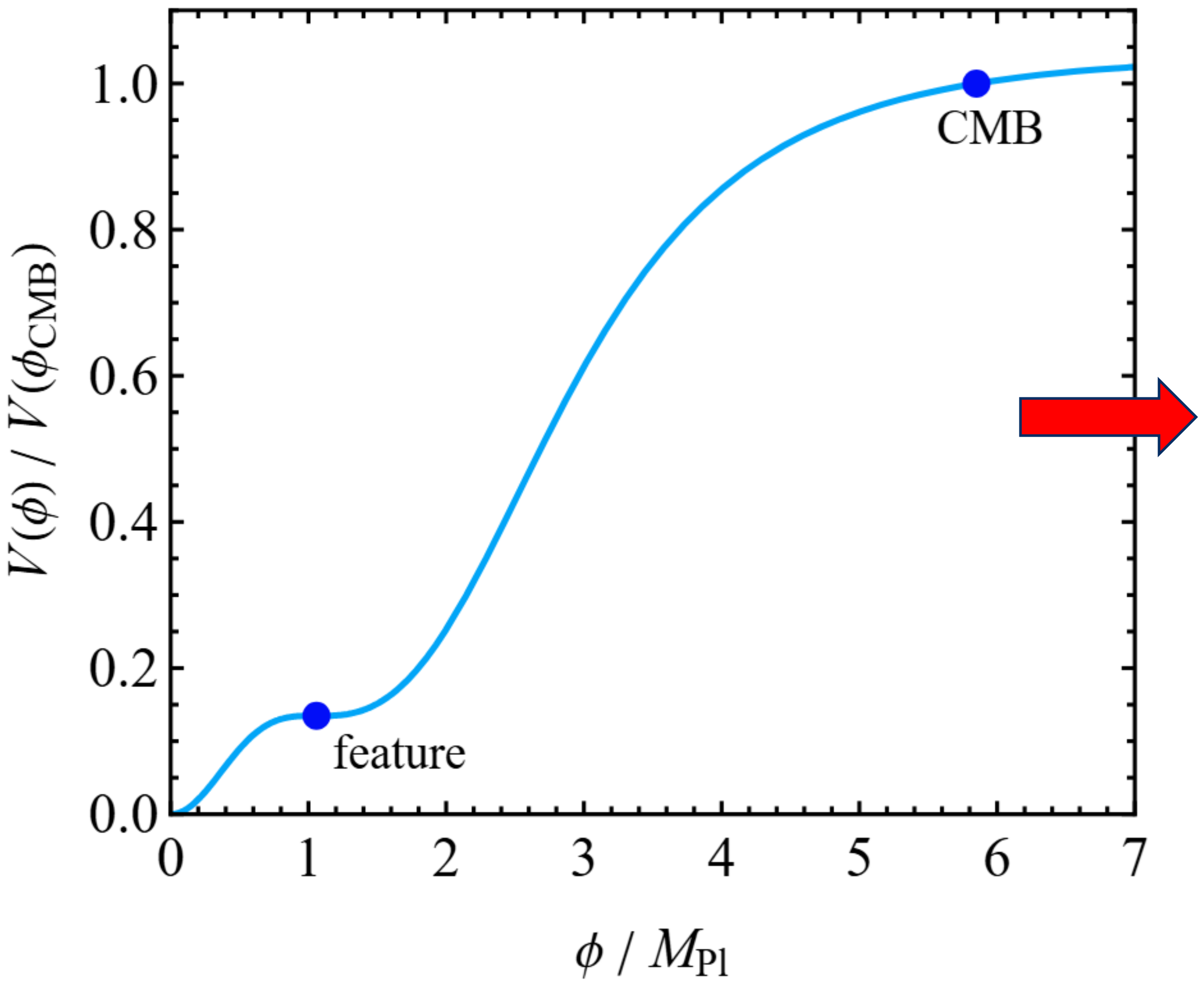


Practical problems

Features in potential on a short field interval

Usually eternal inflation at large field values



Practical problems

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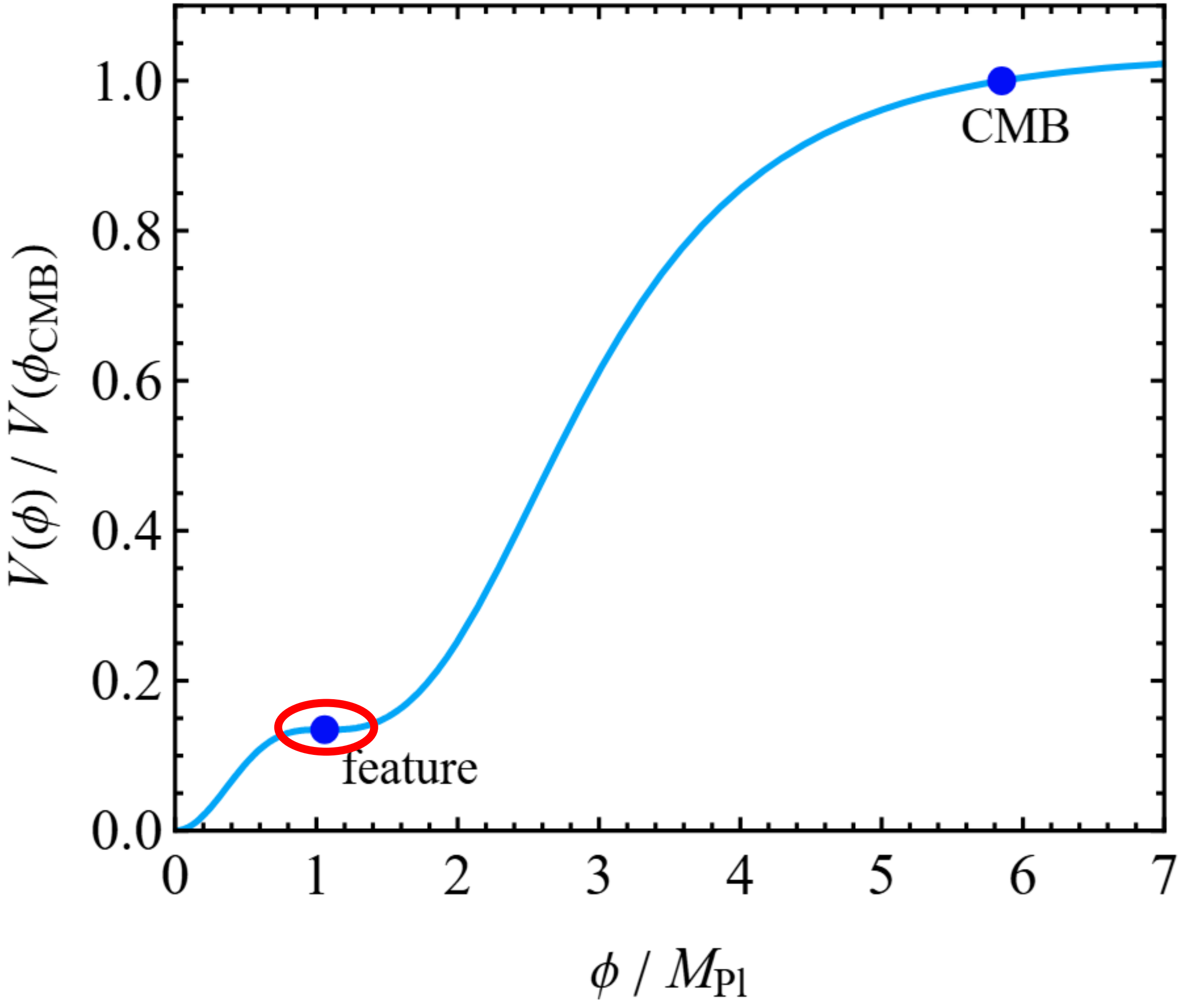
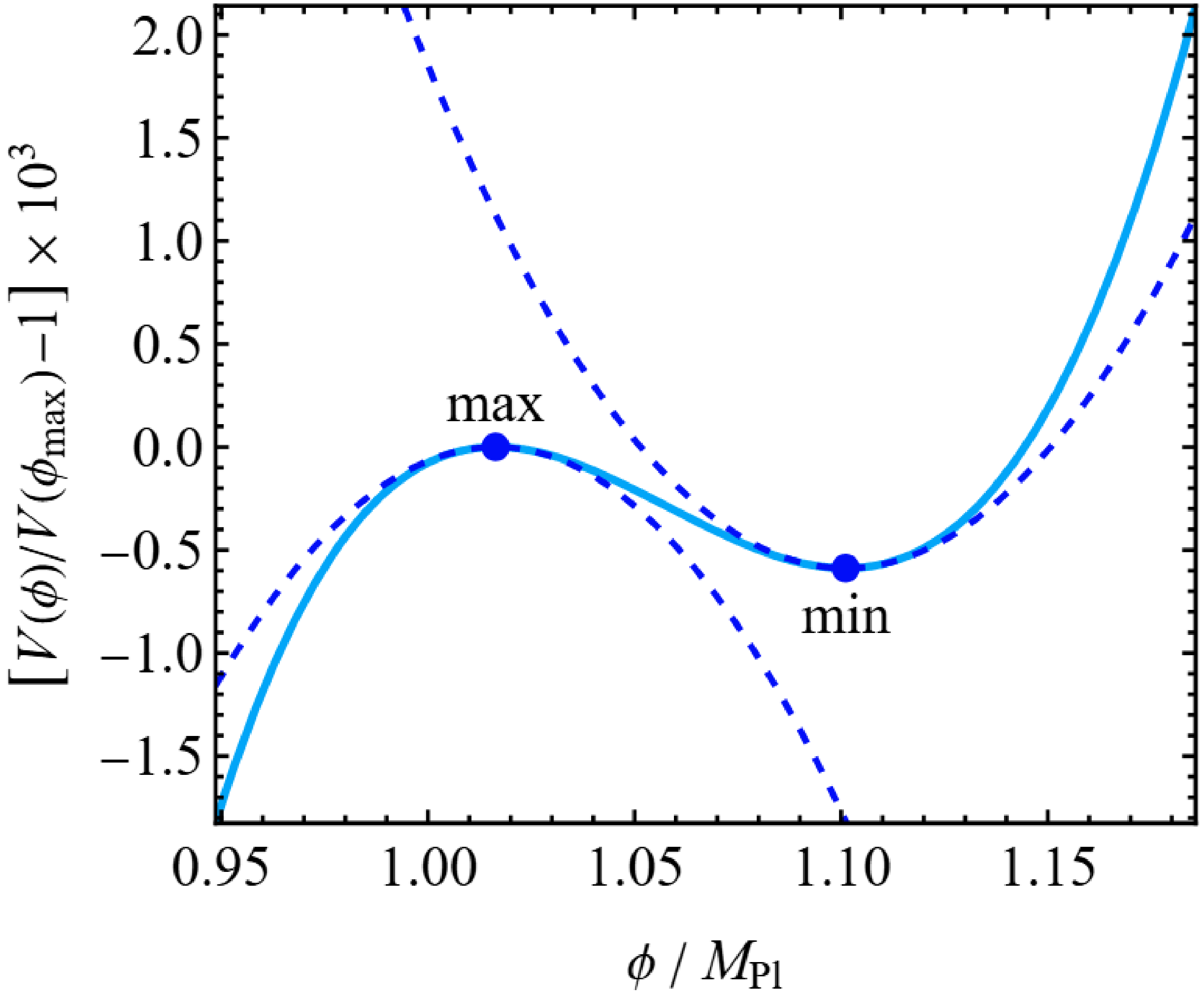
Focus on the feature



Zoom into the feature

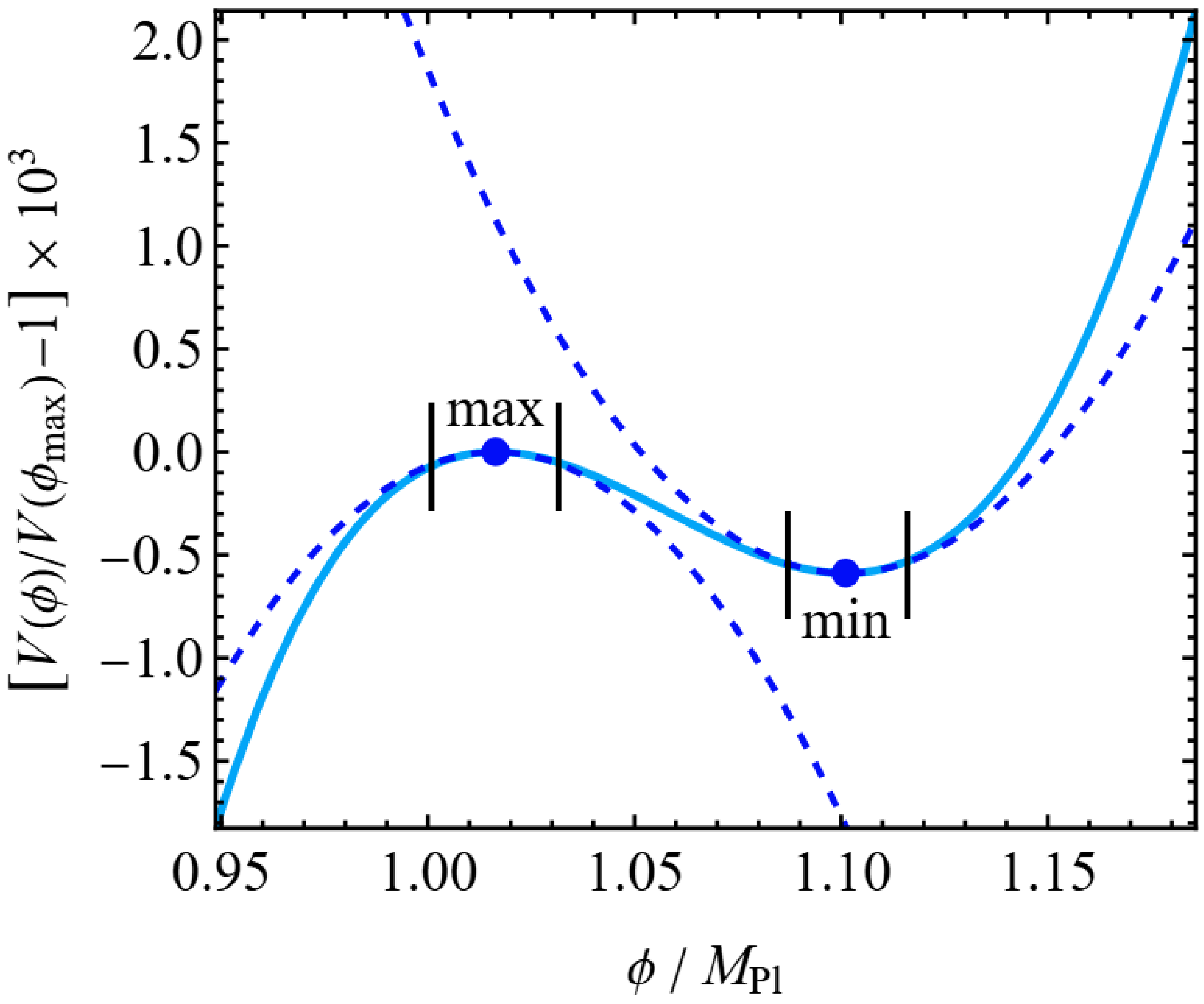


Parabolic approximation:

$$\sigma = \frac{H}{2\pi} \times \frac{\Gamma\left(\frac{3}{2} - \eta_H\right)}{\Gamma\left(\frac{3}{2}\right)} \times \left(\frac{2}{\sigma_c}\right)^{-\eta_H}$$

$$\mu = -\eta_H(\phi - \phi_i)$$

Zoom into the feature



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Absorbing boundaries at $\phi = \phi_i \pm \phi_b$

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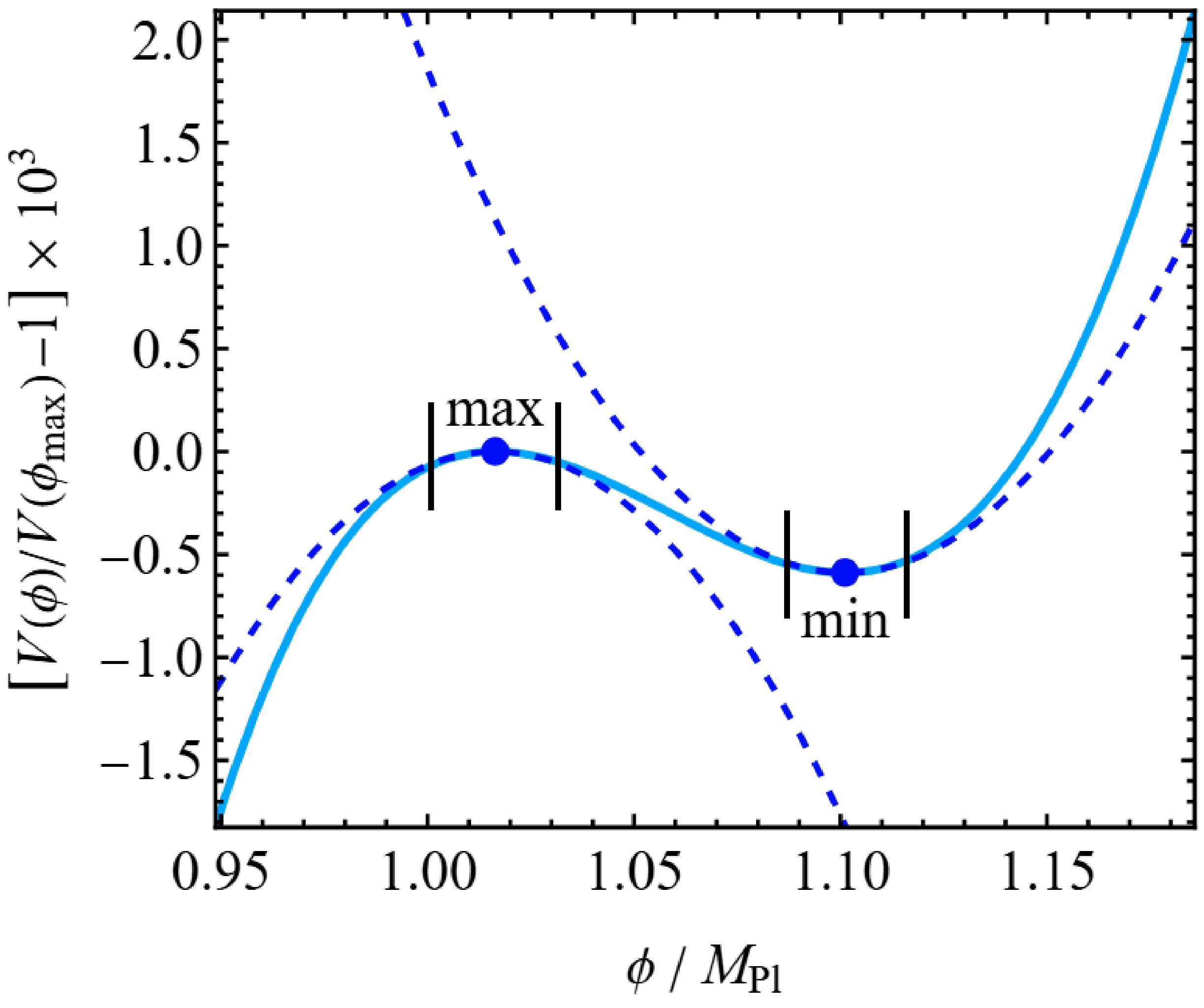

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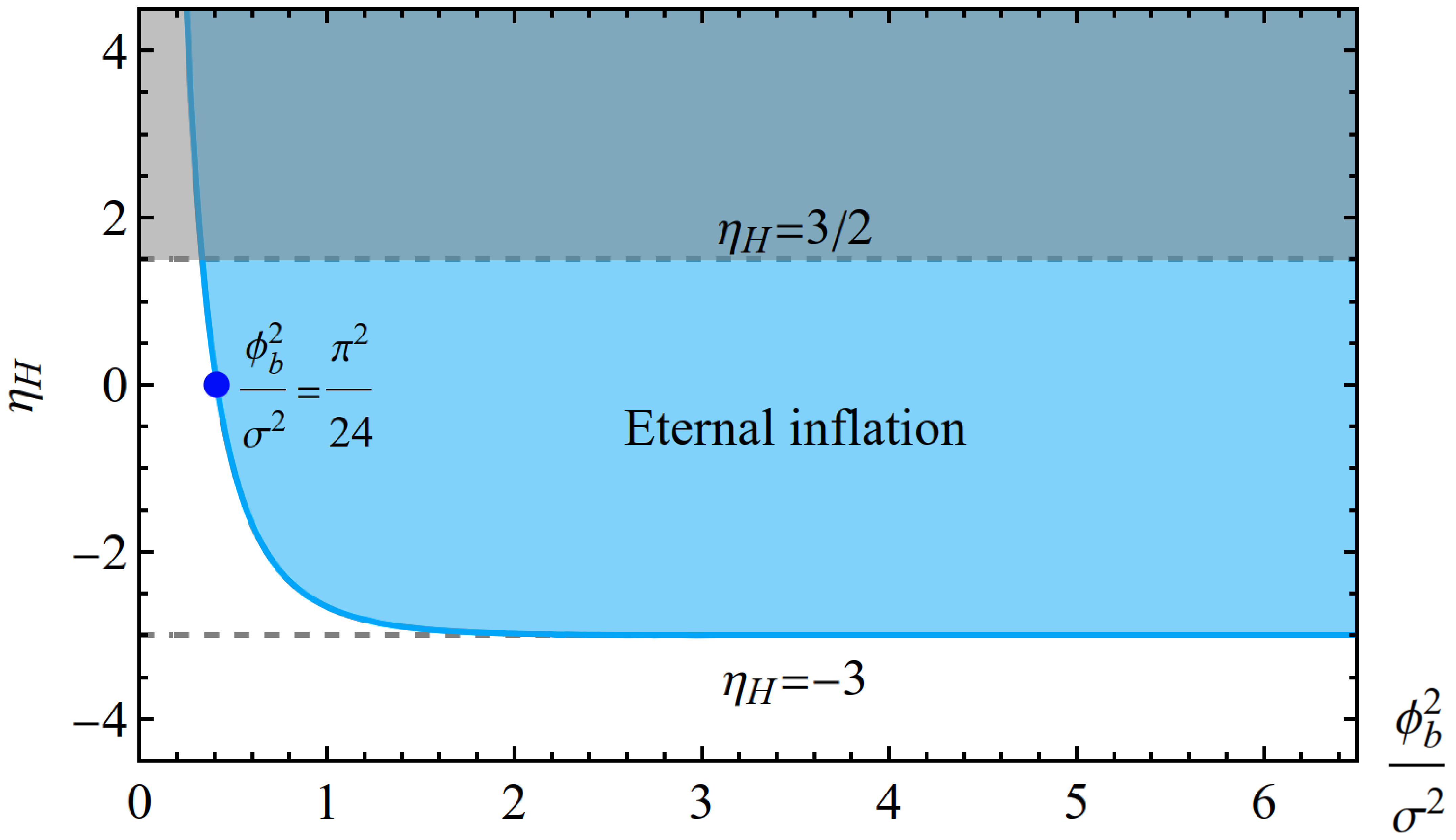
$\eta_H=3/2$
$\frac{\phi_b^2}{\sigma^2}=\frac{\pi^2}{24}$
Eternal inflation
$\eta_H=-3$
η_H
$\frac{\phi_b^2}{\sigma^2}$
4
2
0
−2
−4
0
1
2
3
4
5
6

Wide limit

$$\phi_b \gtrsim \sigma$$

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$\sim M_{\mathrm{Pl}}$ $\qquad\qquad$ $\sim H$

Wide limit

$$\phi_b \gtrsim \sigma$$

$\sim M_{\rm Pl}$ $\qquad\qquad$ $\sim H$

Maximum:

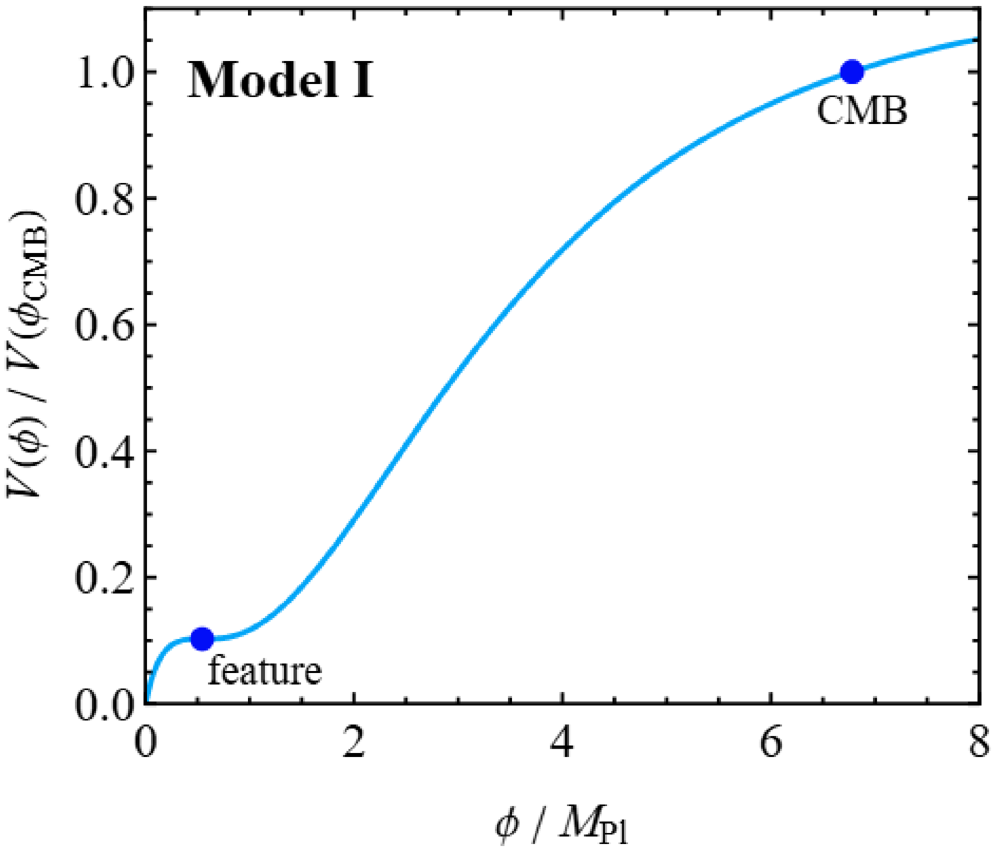
$$\lambda \approx |\eta_H| \sim 0.1$$

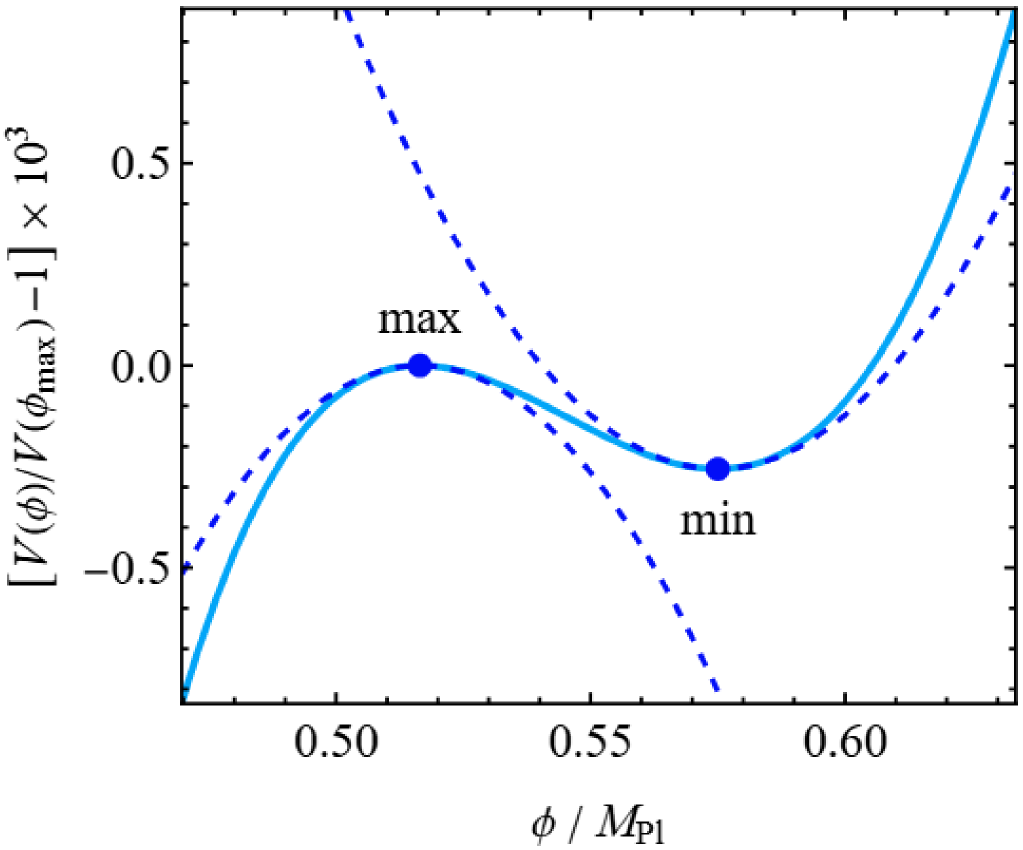
Minimum:

$$\lambda \approx 0$$

Typical potentials:

[Kannike et al. '17]





max: $\phi_b^2/\sigma^2 \sim 10^4\,,\ \lambda = -\eta_H = 0.413$

min: $\phi_b^2/\sigma^2 \sim 10^8\,,\ \lambda \approx 0$

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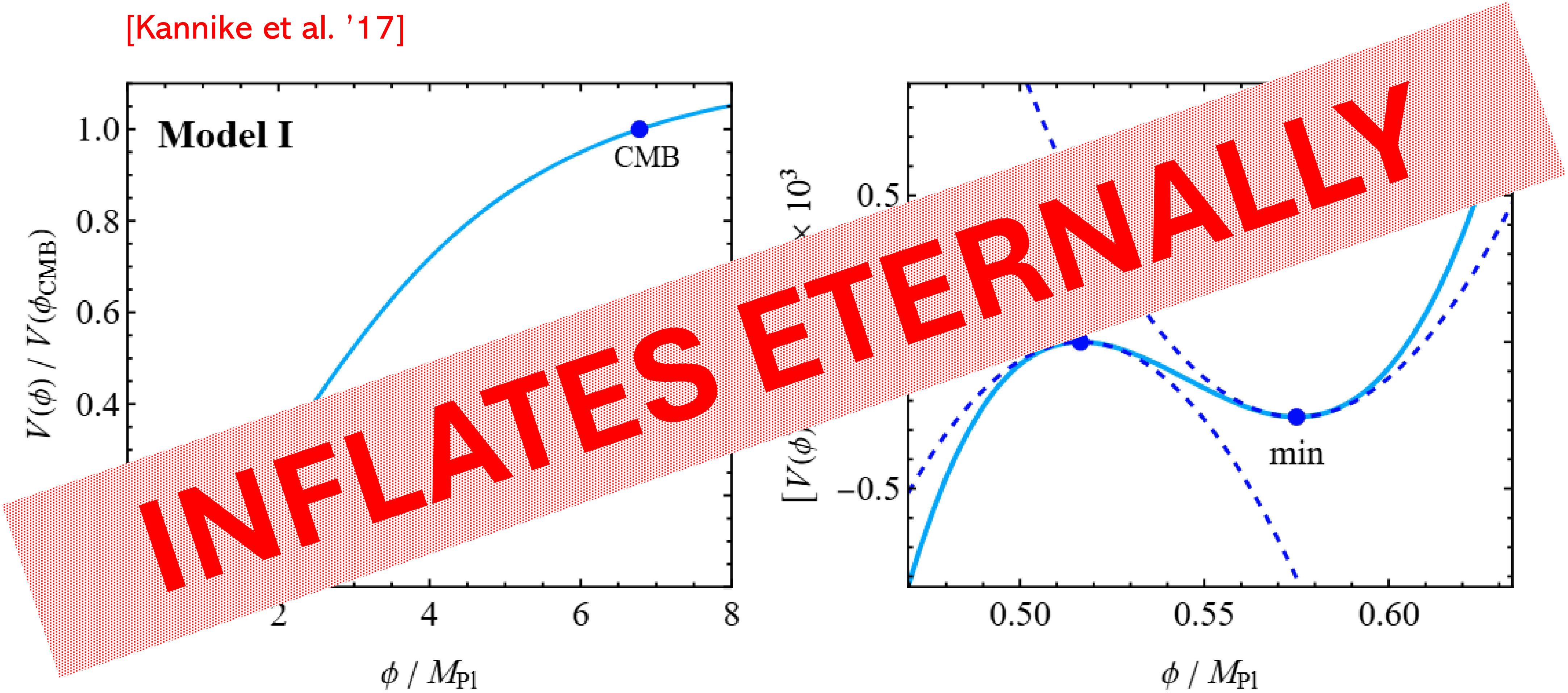

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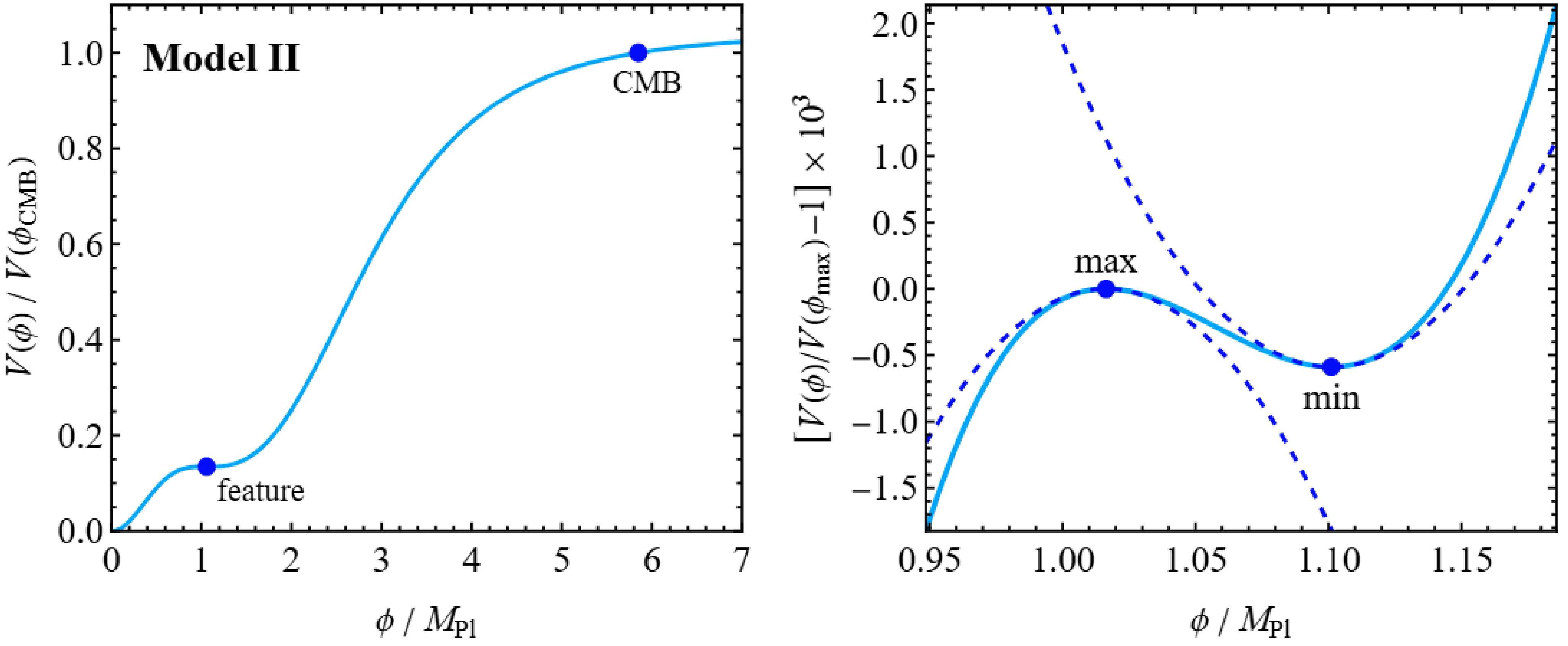
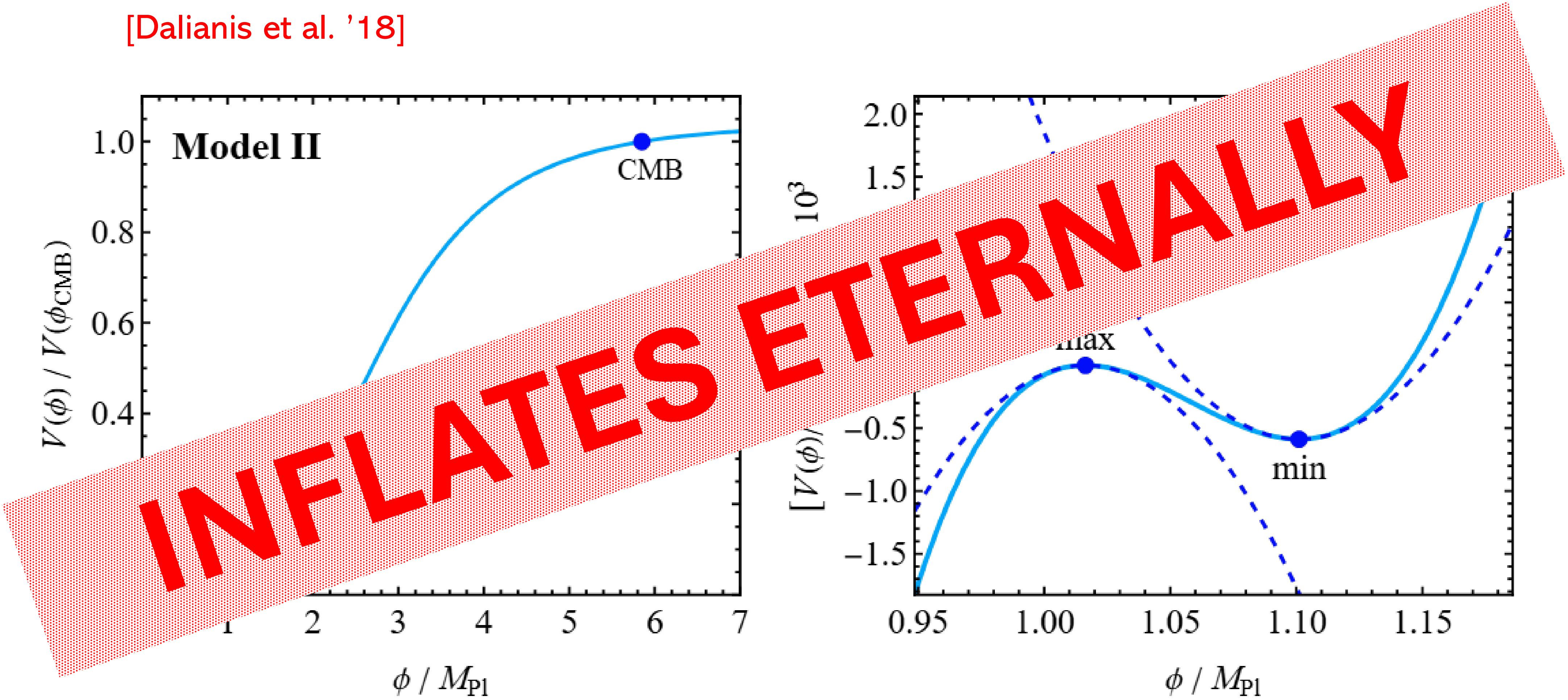


max: $\phi_b^2/\sigma^2 \sim 10^4\,,\ \lambda = -\eta_H = 0.441$

min: $\phi_b^2/\sigma^2 \sim 10^9\,,\ \lambda \approx 0$

Typical potentials:

[Dalianis et al. '18]



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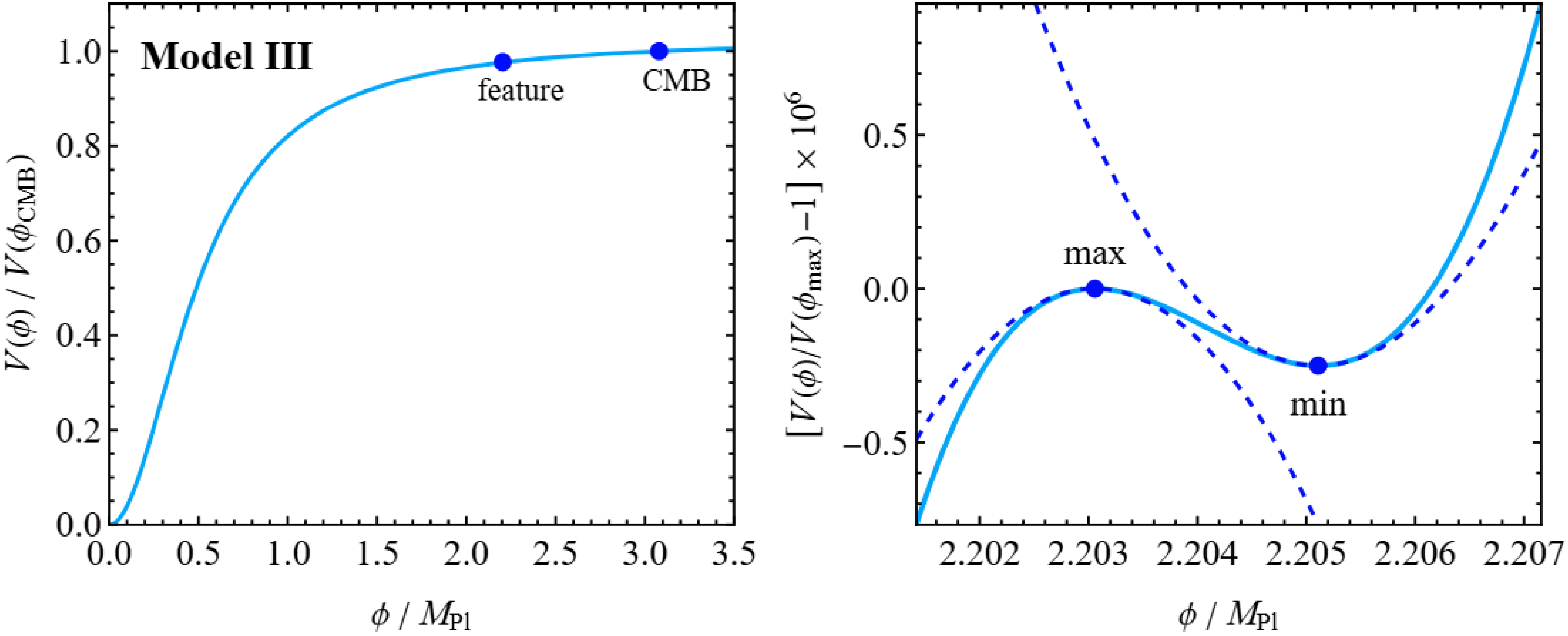
Typical potentials:

[Mishra & Sahni '19]

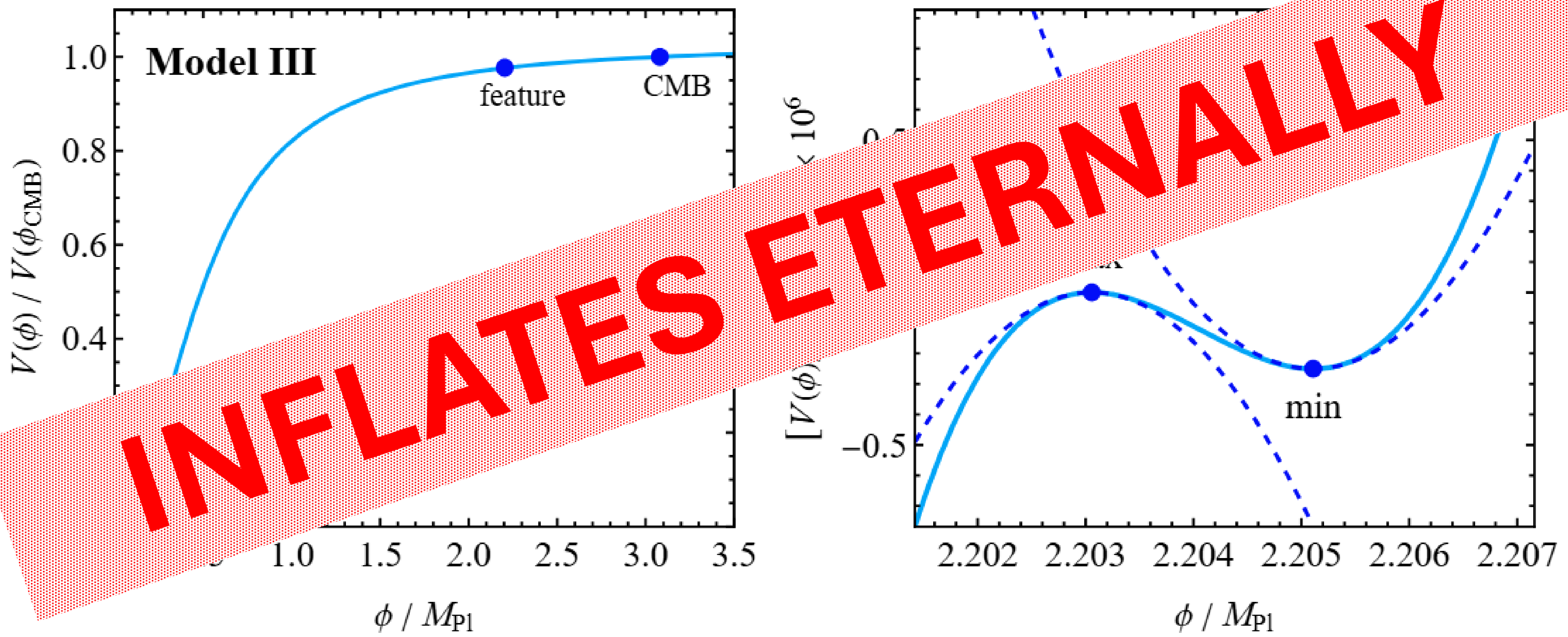


max: $\phi_b^2/\sigma^2 \approx 40\,,\ \lambda = -\eta_H = 0.329$

min: $\phi_b^2/\sigma^2 \sim 10^5\,,\ \lambda \approx 0$

Typical potentials:

[Mishra & Sahni '19]



INFLATES ETERNALLY

max: $\phi_b^2/\sigma^2 \approx 40\,,\ \lambda = -\eta_H = \underline{0.329}$

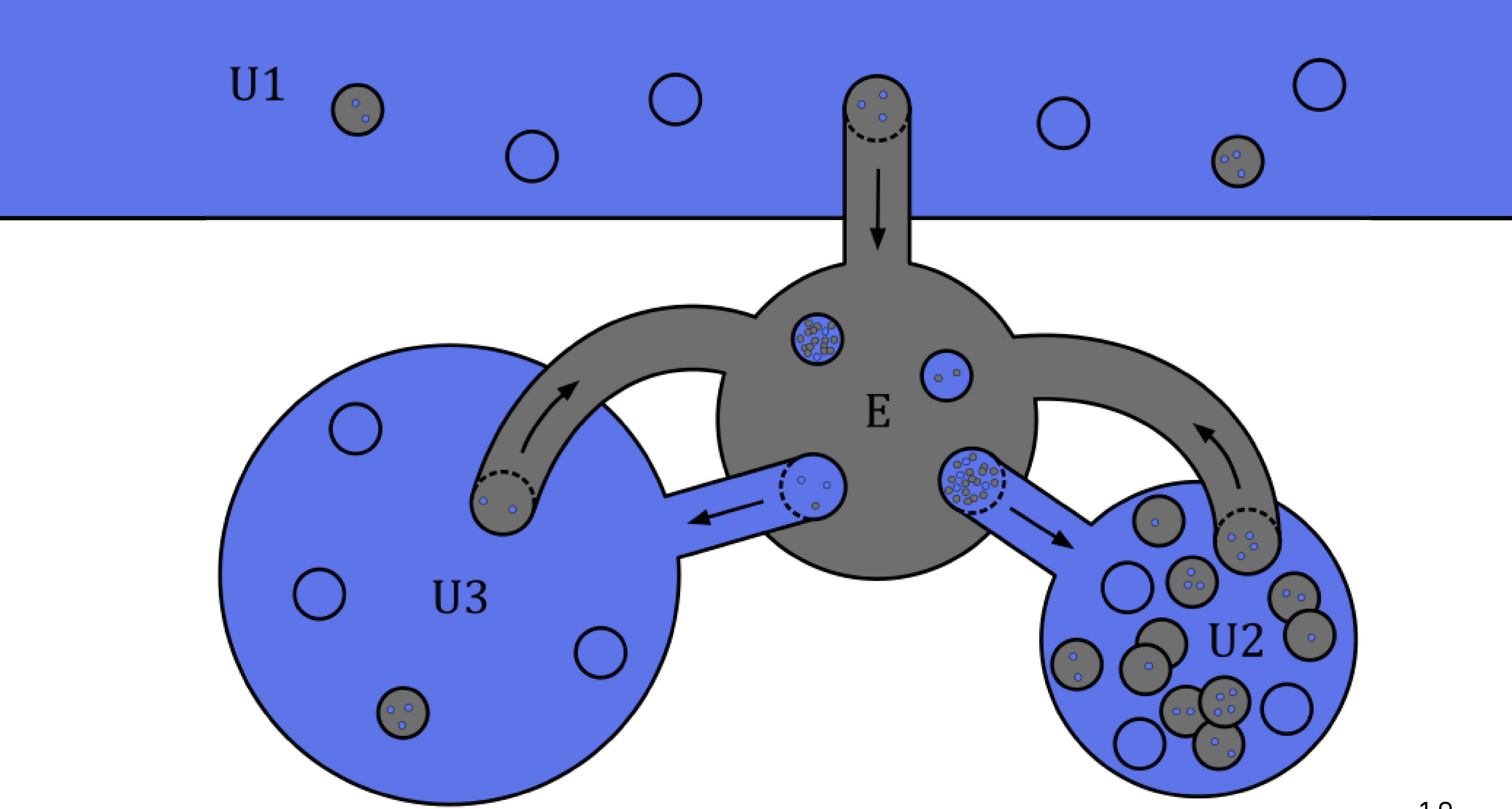
min: $\phi_b^2/\sigma^2 \sim 10^5\,,\ \lambda \approx \underline{0}$

Eternal inlation is generic!

Structure of space-time

Eternal inflation:
inside "extreme" black holes (type II)

What does the Universe look like
inside these black holes?


U1
E
U3
U2

Which universe do we live in?

Volume weighted probabilities:

U1 < U3 (large) < U3 (small) < U2

Which universe do we live in?

Volume weighted probabilities:

Compatible with CMB		Incompatible with CMB
U1 < U3 (large)	<	U3 (small) < U2

Which universe do we live in?

Volume weighted probabilities:

Compatible with CMB

U1 < U3 (large) <

Incompatible with CMB

U3 (small) < U2

Most likely for us

Can't support life?

Eternally inflating PBH models incompatible with the CMB! *

* If volume weighting is used

U1 and U3 (large) still exist,
only with small volume fraction

If volume weighting abandoned: eternal inflation can't solve initial conditions for inflation, either

Eternal inflation is generic in inflection point PBH models

Models incompatible with CMB?

Volume weighting?

Parabolic approximation

Kummer's equation; lowest eigenvalue from

$$ {}_1F_1\left(-\frac{\lambda_1}{2\eta_H};\frac{1}{2};\frac{\phi_b^2}{\sigma^2}\eta_H\right)=0 $$

Wide limit: $\phi_b^2 \gtrsim \sigma^2 \implies \lambda_1 \approx \begin{cases} |\eta_H|\,, & \eta_H<0 \\ 0\,, & \eta_H>0 \end{cases}$

$$\eta_H = \frac{3}{2}\left(1 - \sqrt{1 - \frac{4}{3}\eta_V}\right)$$